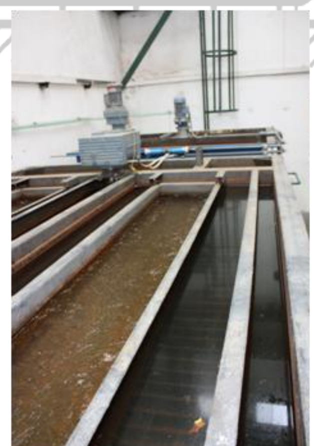
# SETTLING AND SEDIMETATION IN PARTICLE – FLUID SEPARATION

In settling and sedimentation, the particles are separated from the fluid by gravitational forces acting on the particles. Force acting on particle is proportional to particle volume and density difference. For this reason sedimentation can be also used for particle classification by size or by density.

***Particles:***
- solid particles
- liquid drops

***Fluid*** may be at rest or in motion***:***
- liquid
- gas

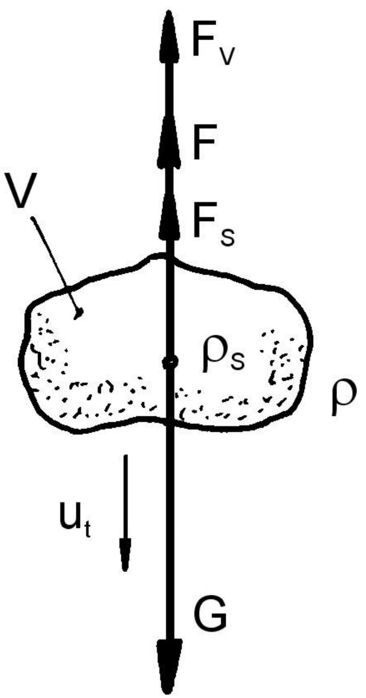
***Industrial applications:***
- removal of solids form from waste water
- settling of crystals from the mother liqour
- separation of liquid-liquid mixture from a solvent-extraction stage
- settling of solid food particles from a liquid food




- Geankopolis, C. J.: ***Transport Processes and Separation Process Principles.*** 4th edition. New Jersey: Publishing as Prentice Hall PTR, 2003.1026 p. ISBN 0-13-101367-X.
- Foust, A. S. et al.: ***Principles of Unit Operations.*** New York: Jon Wiley & Sons, Inc., 1960. 578 p.
- Brown, G. G. et. al.: ***Unit Operations. 6th printing.*** New York: Jon Wiley & Sons, Inc., 1956. 611 p.

# *Particle movement through fluid due to gravitation field*

## *Forces acting on particles*




$$G - F_V - F_s - F = 0$$

$$G = V\rho_s g$$

Gravitation force (external force)

$$F_V = V\rho g$$

Buoyant force

$$F_s = V\rho_s \frac{\mathrm{d}u_t}{\mathrm{d}t}$$

Inertial force – due to acceleration

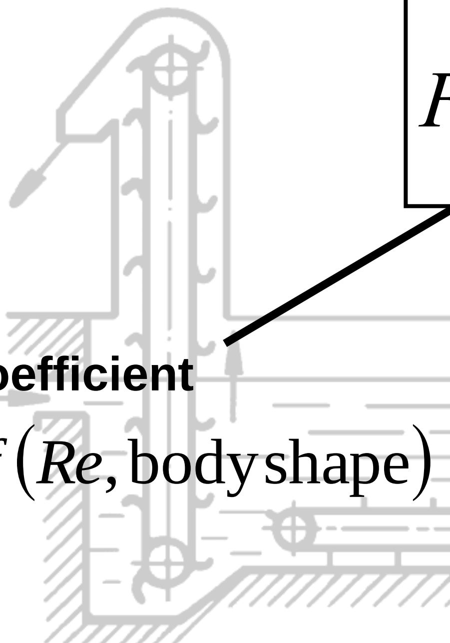
$$F = C_{Dt} S_p \frac{u_t^2}{2} \rho$$

Drag force

# *Fluid resistance – Drag force F*

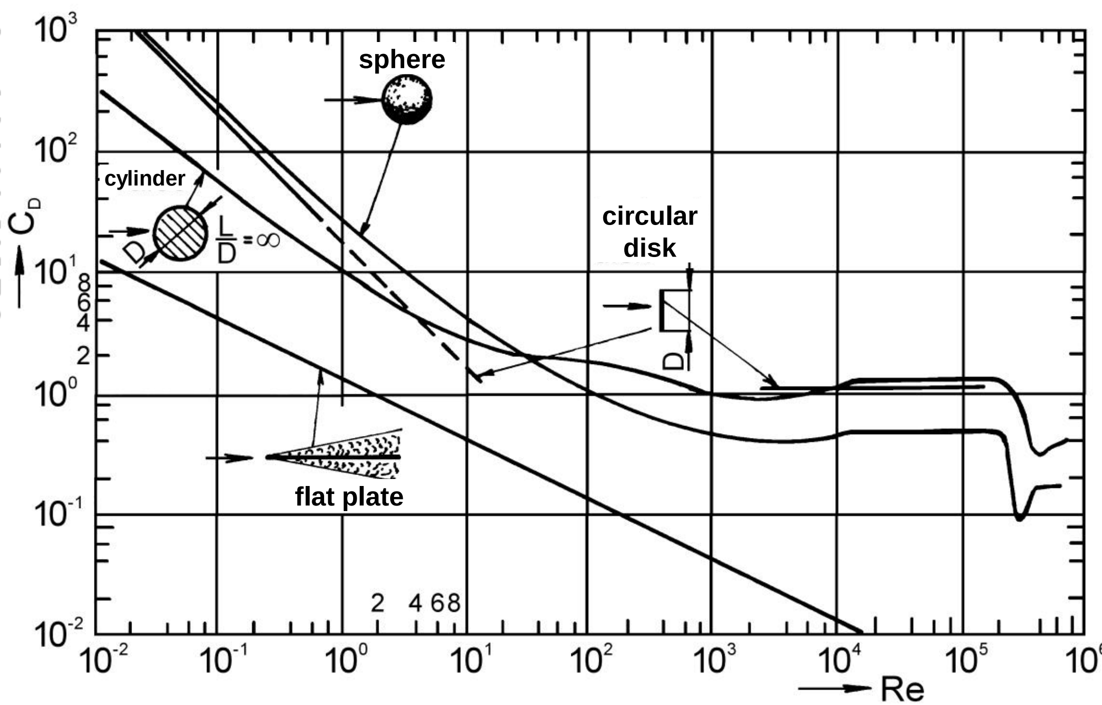
- ***Skin friction*** – due to viscous friction of fluid on the surface of the body
- ***Form drag*** – dynamic pressure force acting on the surface of the body

$$F = C_D S_p \frac{u^2}{2} \rho$$



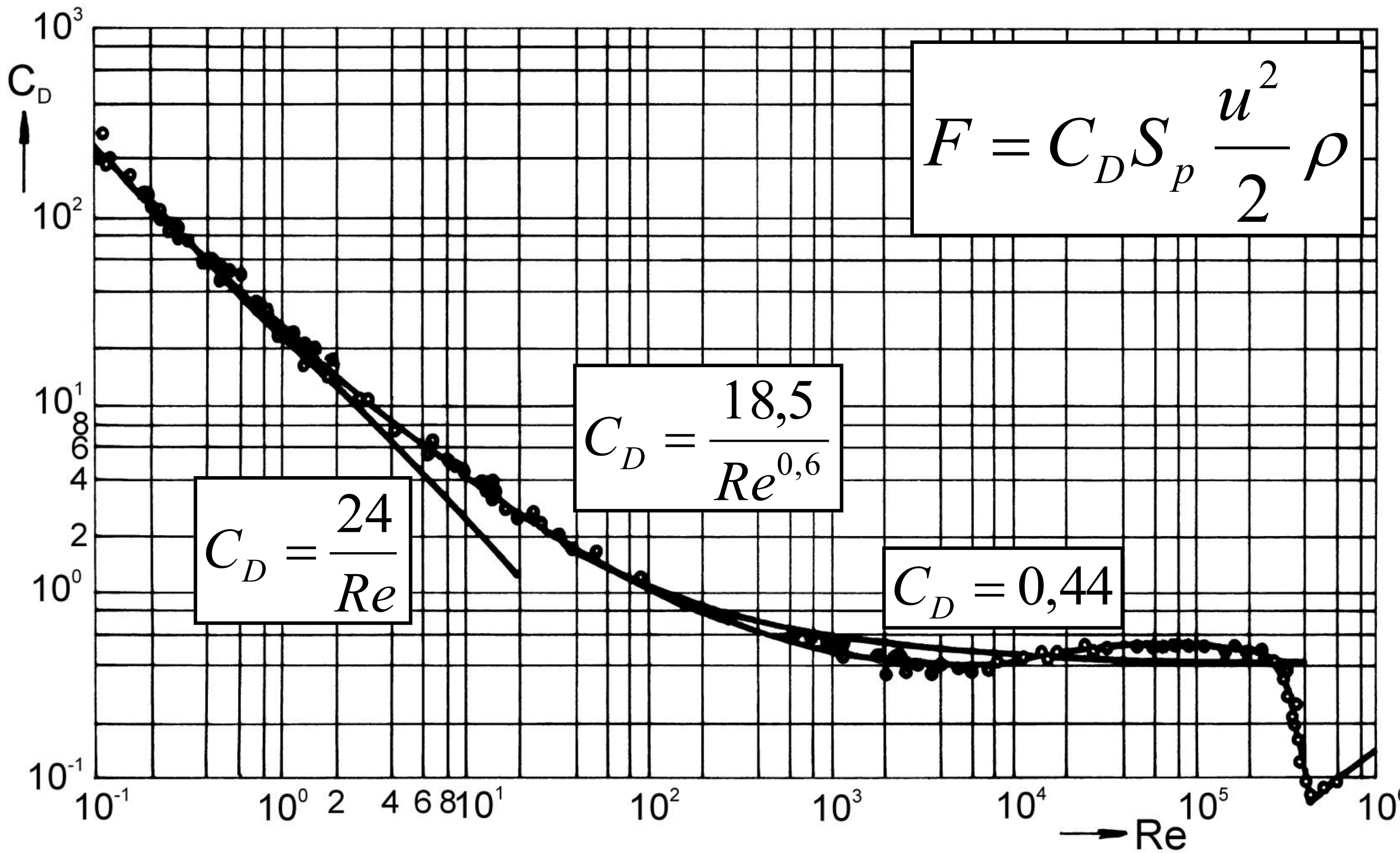
**Drag coefficient**

$$C_D = f\left(Re, \text{body shape}\right)$$

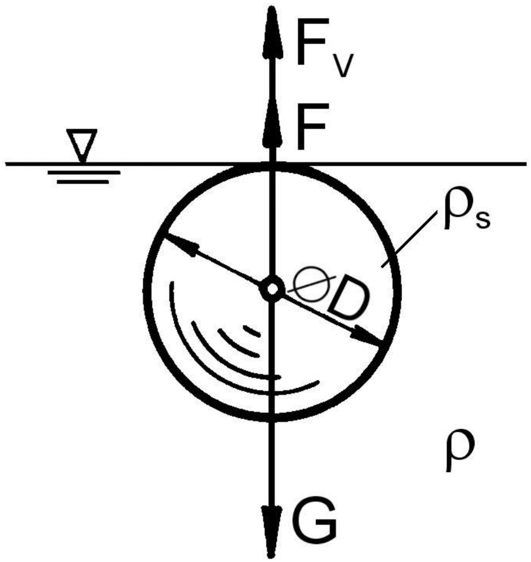


# ***Drag coefficient for rigid sphere***

| ***Stokes region*** | ***Transition region*** | **Newton region** |
|---|---|---|
| $Re \leq 2$ | $2 < Re < 500$ | $500 < Re < 3 \cdot 10^5$ |



# *Stationary particle movement – free settling velocity*




$$G - F_V - \cancel{F_s} - F = 0$$

$$\frac{\pi D^3}{6}\rho_s g - \frac{\pi D^3}{6}\rho g - C_D \frac{\pi D^2}{4}\frac{u^2}{2}\rho = 0$$

$$u = \sqrt{\frac{4}{3}\frac{D(\rho_s - \rho)g}{C_D \rho}}$$

***Stokes region*** ($Re \le 2$): $$u = \frac{D^2(\rho_s - \rho)g}{18\mu}$$

***Transition region*** ($2 < Re < 500$): $$u = 0{,}153\,\frac{D^{1,14}(\rho_s - \rho)^{0,71} g^{0,71}}{\rho^{0,29}\mu^{0,43}}$$

***Newton region*** ($500 < Re < 3\cdot 10^5$): $$u = 1{,}74\sqrt{\frac{D(\rho_s - \rho)g}{\rho}}$$

# Calculation of settling velocity

***Settling velocity can not be calculated directly***, because we don't know region of settling. This region is determined by the value of Reynolds number, where contains unknown velocity *u*. For this reason, we will find a new criterion free settling velocity:

$$u = \sqrt{\frac{4}{3}\frac{D(\rho_s - \rho)g}{C_D \rho}} \quad \Rightarrow \quad C_D = \frac{4}{3}\frac{D(\rho_s - \rho)g}{u^2 \rho},$$

where $C_D = f(Re)$

$$Re = \frac{uD\rho}{\mu}.$$

Both criterions contain unknown settling velocity. New criterion can be determined as multiple of $C_D Re^2$:

$$C_D Re^2 = \frac{4}{3}\frac{D(\rho_s - \rho)g}{u^2 \rho}\frac{u^2 D^2 \rho^2}{\mu^2} = \frac{4}{3}\frac{D^3(\rho_s - \rho)\rho g}{\mu^2}$$

Value of this new criterion can be calculated from known parameters. On the basis of this value we can decide on region of settling.

Boundary values of this criterion can be calculated from limit values of *Re*:

For ***Stokes*** region ($Re < 2$, $C_D = 24/Re$) we get: $\mathbf{C_D Re^2 < 48}$
For ***transition*** region valid: $\mathbf{48 < C_D Re^2 < 1.1 \cdot 10^5}$
For ***Newton*** regime ($500 < Re < 3 \cdot 10^5$, $C_D = 0.44$) we get: $\mathbf{1.1 \cdot 10^5 < C_D Re^2 < 4 \cdot 10^{10}}$

# Calculation of particle diameter from settling velocity

***Particle diameter can not be also calculated directly***, because analogous to previous case unknown diameter is contained in dimensionless drag coefficient $C_D$ and Reynolds number *Re*. New dimensionless criterion devoid of particle diameter can be obtained in this form:

$$\frac{C_D}{Re} = \frac{4}{3}\frac{D(\rho_s - \rho)g}{u^2\rho}\frac{\mu}{uD\rho} = \frac{4}{3}\frac{(\rho_s - \rho)g\mu}{u^3\rho^2}$$

Boundary values of this criterion can be calculated from limit values of *Re*:

For ***Stokes*** region is **$C_D/Re > 6$**
For ***transition*** region is **$8.8\cdot10^{-4} < C_D/Re < 6$**
For ***Newton*** region is **$1.47\cdot10^{-6} < C_D/Re < 8.8\cdot10^{-4}$**

Determination of particle diameter from settling velocity:

***Stokes region*** ($Re \le 2$):

$$D = 3\cdot\sqrt{\frac{2u\mu}{(\rho_s - \rho)g}}$$

***Transition region*** ($2 < Re < 500$):

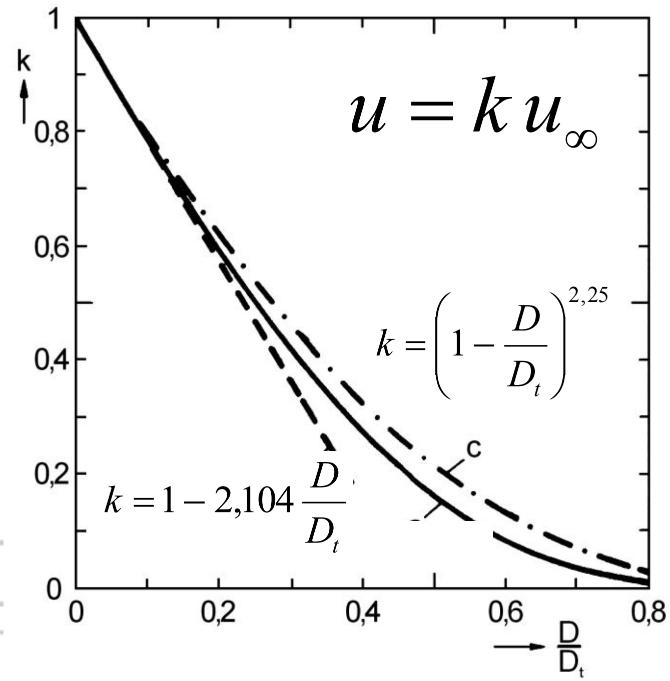
$$D = 5{,}19\,\frac{u^{0,877}\rho^{0,254}\mu^{0,377}}{(\rho_s - \rho)^{0,623}g^{0,623}}$$

***Newton region*** ($500 < Re < 3\cdot10^5$):

$$D = 0{,}33\,\frac{u^2\rho}{(\rho_s - \rho)g}$$

# Hindered settling

- ***Wall effect*** →




- ***Effect of electrical forces***

  At particles smaller than 100 μm, for particles smaller than 1μm (colloids) electrical separation forces stop sedimentation – it is necessary to change electrical charge by appropriate additive (electrolyte – coagulation, polymer – flocculation). By this way greater better settling agglomerates are created.

- ***Settling in dilute gasses***

  If mean trajectory of molecules is comparable to particle size. Settling velocity can be determined from empirical formulas.

- ***Effect of fluid motion***

  *Laminar flow* – no influence

  *Turbulent flow* – decreases critical value of *Re* and settling velocity for small particles comparable with size of turbulent vortexes.

- ***Interactions between particles in slurry***

$$u = u_\infty \varphi(\varepsilon)$$

Particles are crowded ⇒ decrease their settling velocity

Equation for determination of hindered settling velocity by Goroško, Rozenbaum a Todes:

$$Re = \frac{Ar\varepsilon^{4,75}}{18 + 0,6\sqrt{Ar\varepsilon^{4,75}}}, \quad \text{where} \quad Ar = \frac{D^3 \Delta\rho\, \rho\, g}{\mu^2}$$

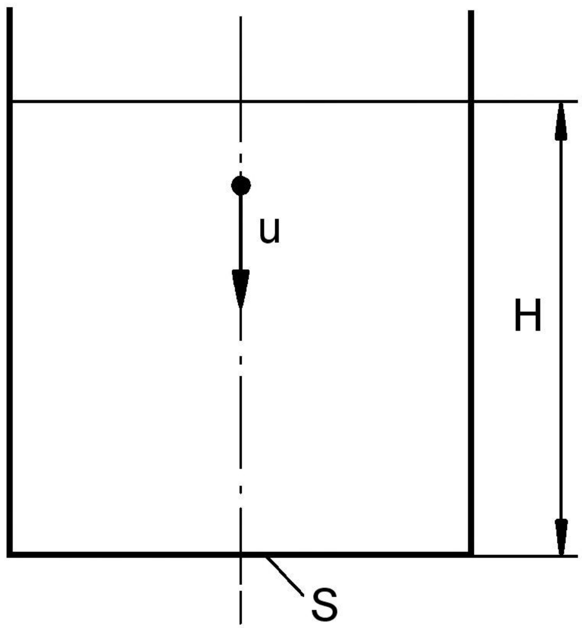
$$\varepsilon \le 0,7: \qquad \varphi(\varepsilon) = 0,123\,\frac{\varepsilon^3}{1-\varepsilon}$$

$$\varepsilon > 0,7: \qquad \varphi(\varepsilon) = \varepsilon^2 \cdot 10^{-1.82(1-\varepsilon)}$$

# Settlers, thickeners

## Batch settling

### *Simple batch settling*




$$t = \frac{H}{u}$$

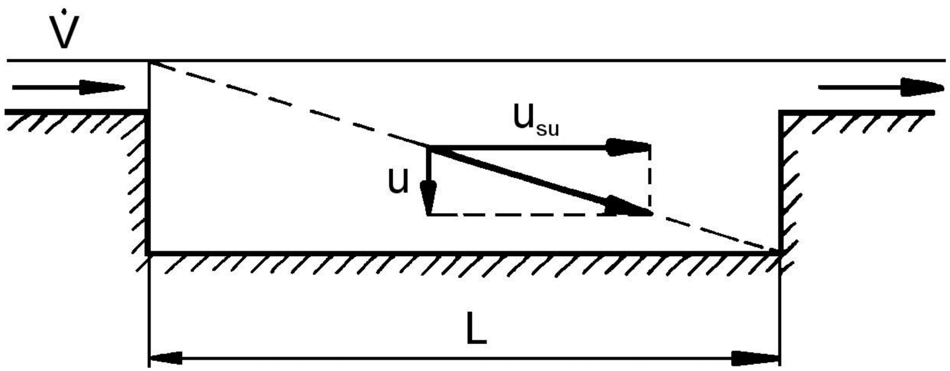
Volumetric capacity of settler

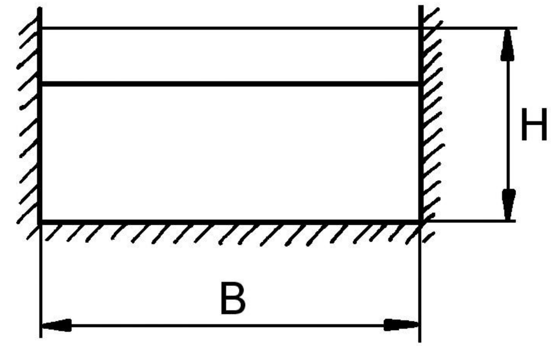
$$\dot{V} = \frac{V}{t} = \frac{SH}{\frac{H}{u}} = Su$$

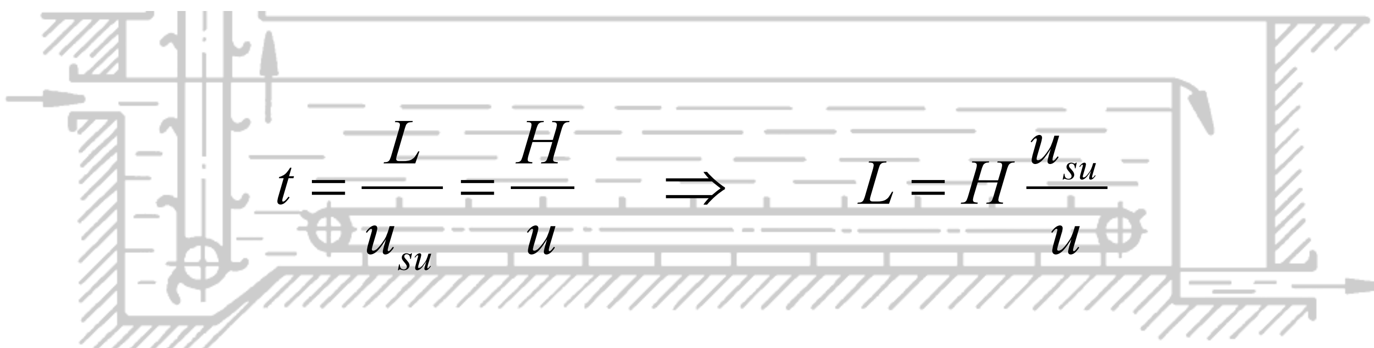
$$\dot{V}_{stř} = \frac{V}{t + t_m}$$

# Semi-continuous settling

## Rectangular settler



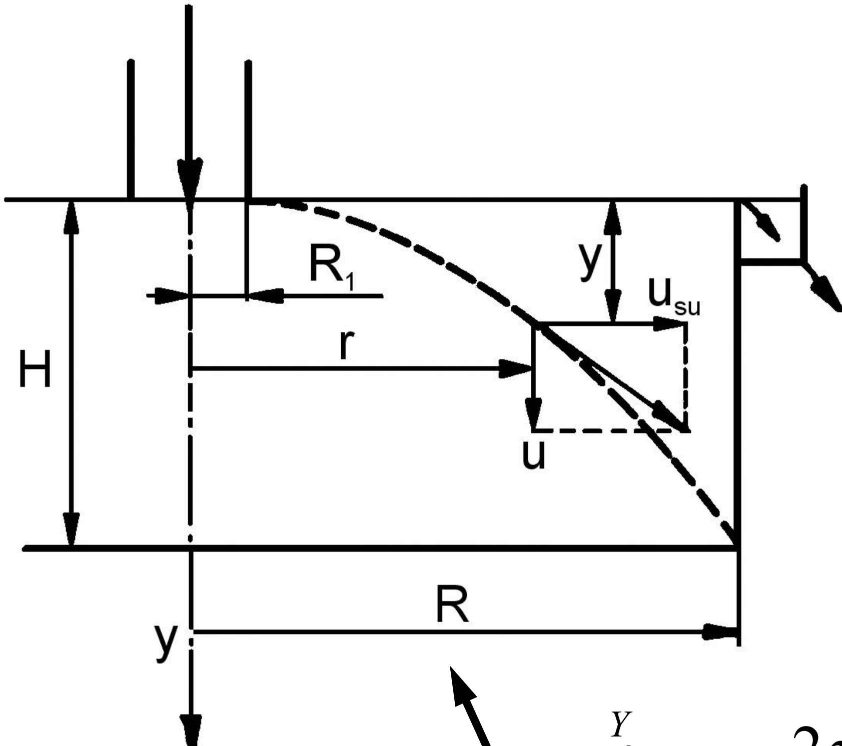



$$t = \frac{L}{u_{su}} = \frac{H}{u} \quad \Rightarrow \quad L = H\frac{u_{su}}{u}$$

Volumetric capacity of settler

$$\dot{V} = BHu_{su} = BLu = Su$$

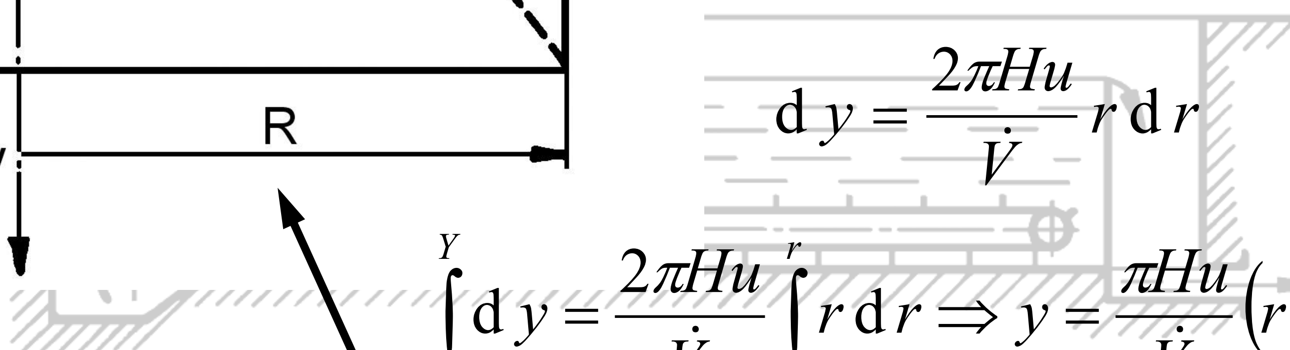
## *Circular settler*




$$\mathrm{d}y = u\,\mathrm{d}t,\ \mathrm{d}r = u_{su}\,\mathrm{d}t$$

$$\mathrm{d}t = \frac{\mathrm{d}y}{\mathrm{d}r} = \frac{u}{u_{su}}$$

$$u_{su} = \frac{\dot{V}}{2\pi r H}$$



$$\mathrm{d}y = \frac{2\pi H u}{\dot{V}} r\,\mathrm{d}r$$

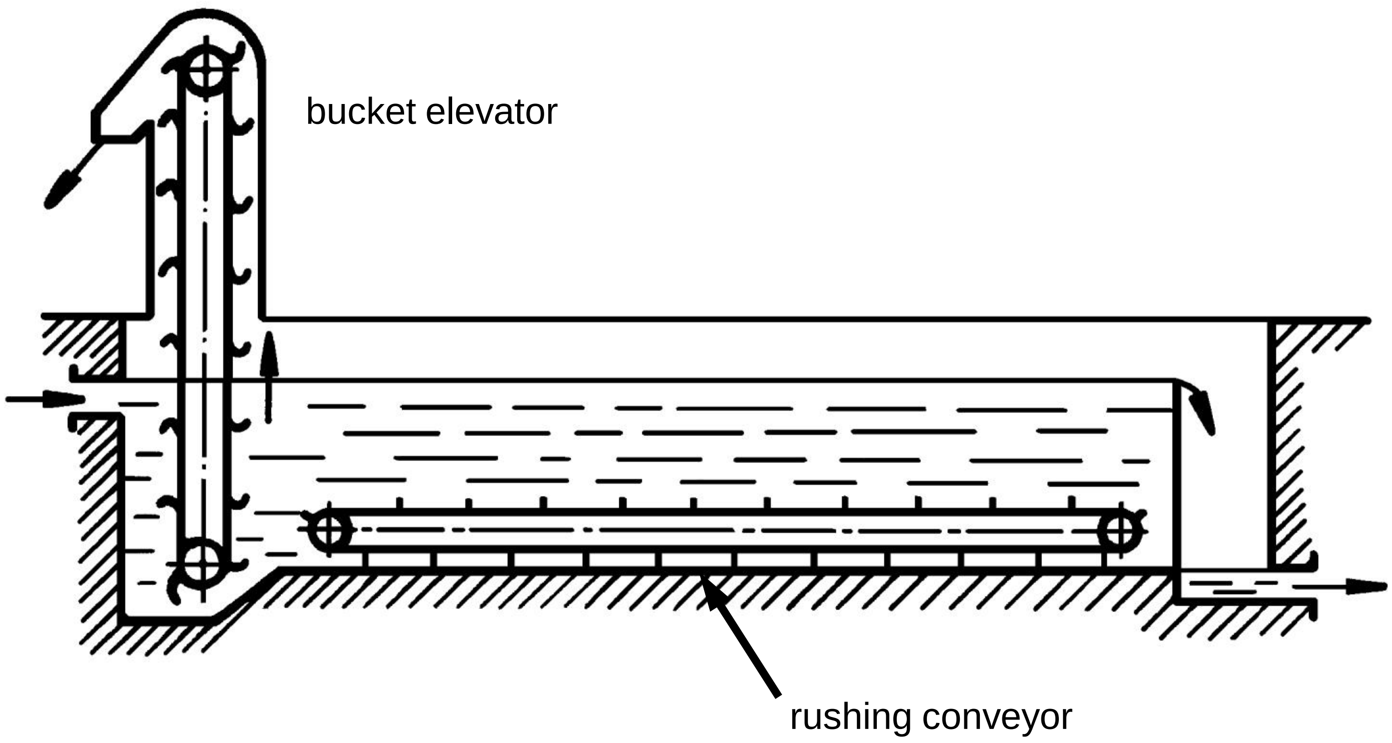
$$\int_0^Y \mathrm{d}y = \frac{2\pi H u}{\dot{V}} \int_{R_1}^{r} r\,\mathrm{d}r \Rightarrow y = \frac{\pi H u}{\dot{V}}\left(r^2 - R_1^2\right)$$

$$y = H, r = R$$

$$R = \sqrt{R_1^2 + \frac{\dot{V}}{\pi u}} \Rightarrow \dot{V} = \pi\left(R^2 - R_1^2\right)u$$

# Continuous settling

## Continuous rectangular settler – sand trap



## *Circular thickener with continuous sludge removal*

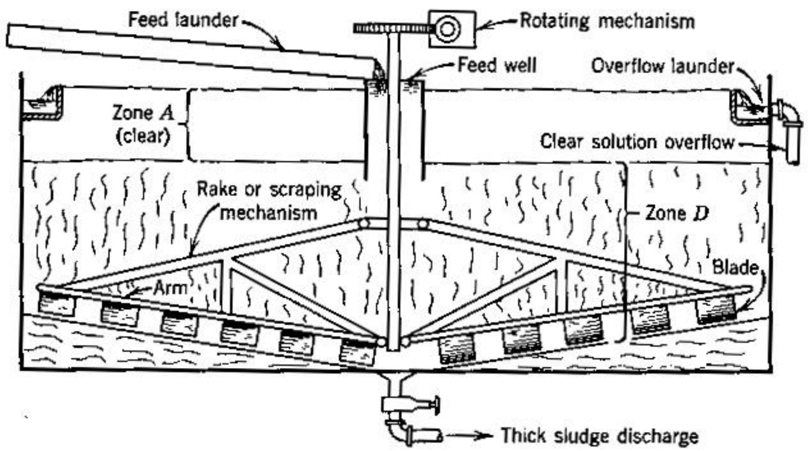
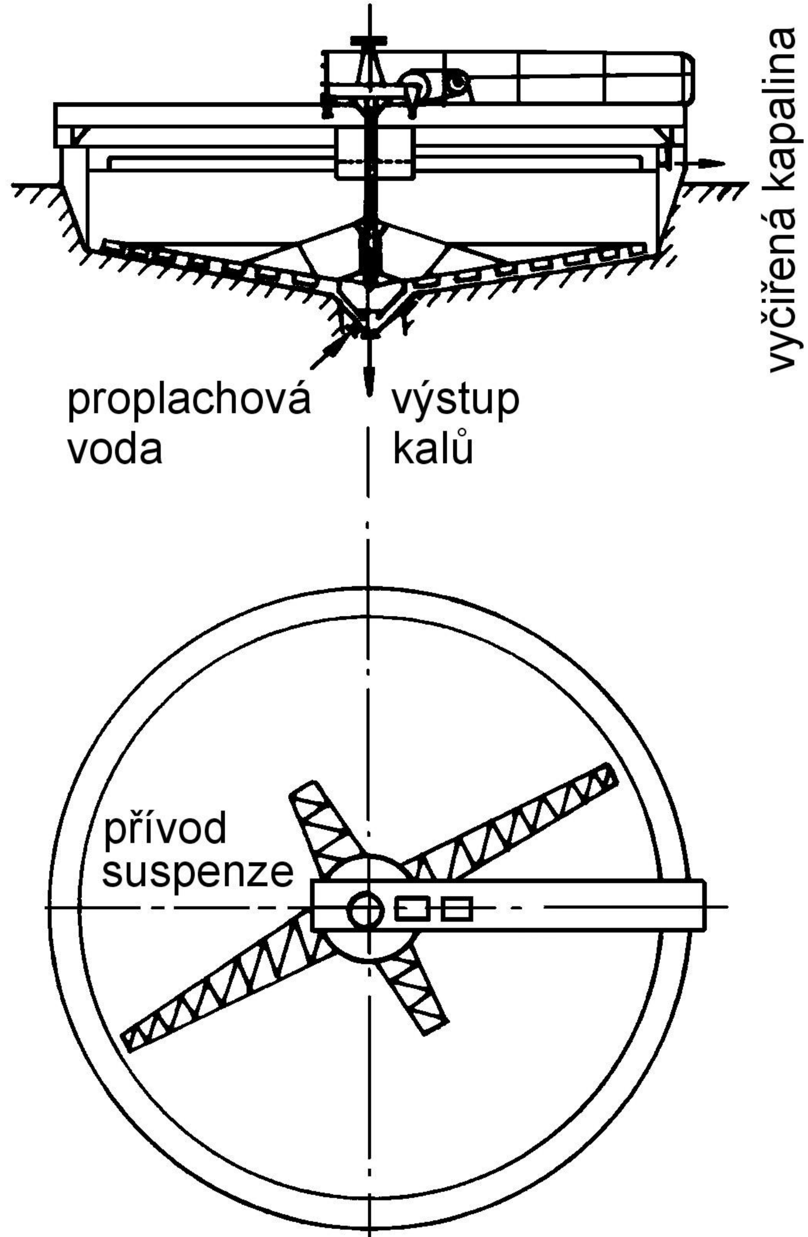
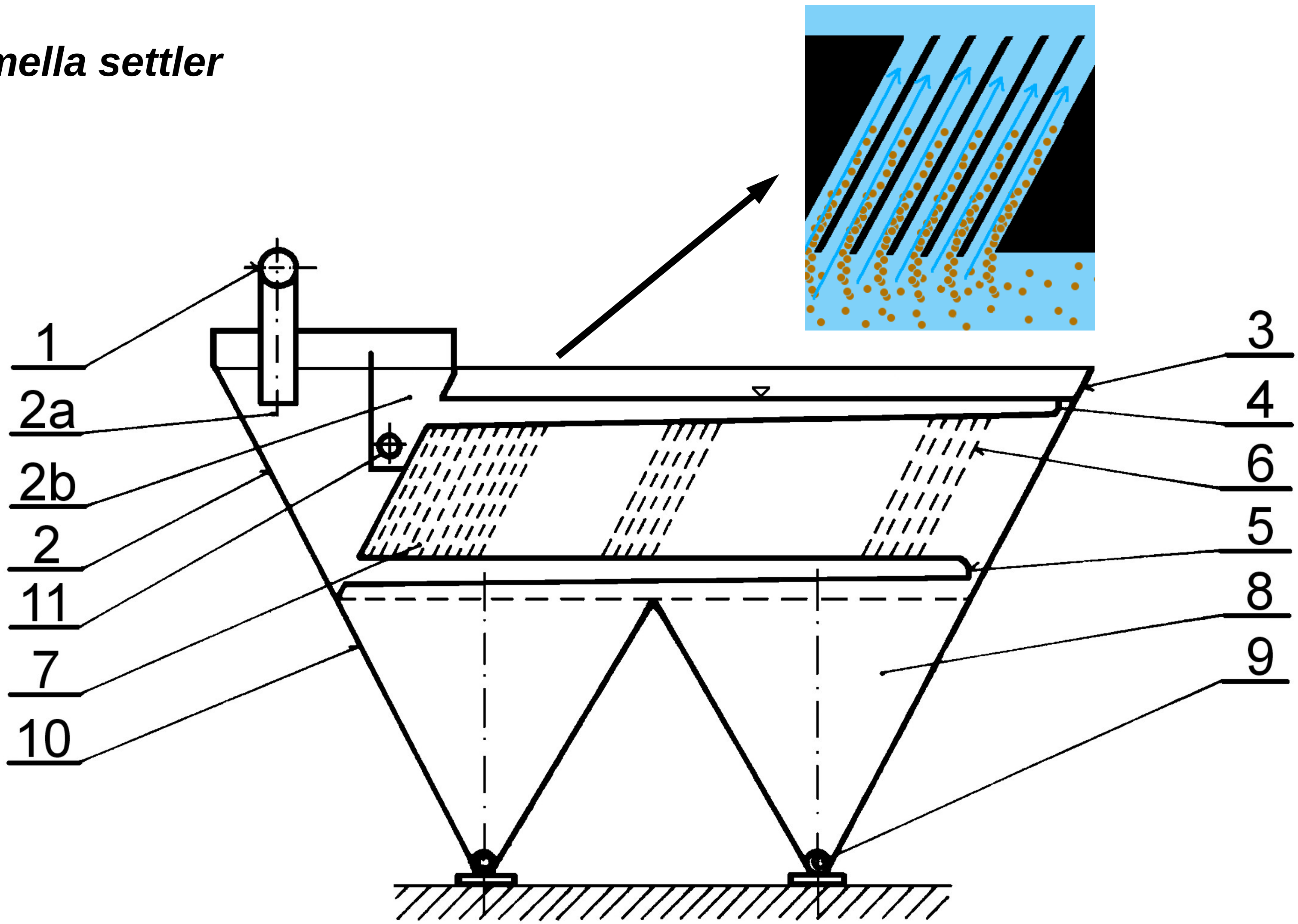



FIG. 101. Sectional diagrammatic drawing of a continuous thickener.



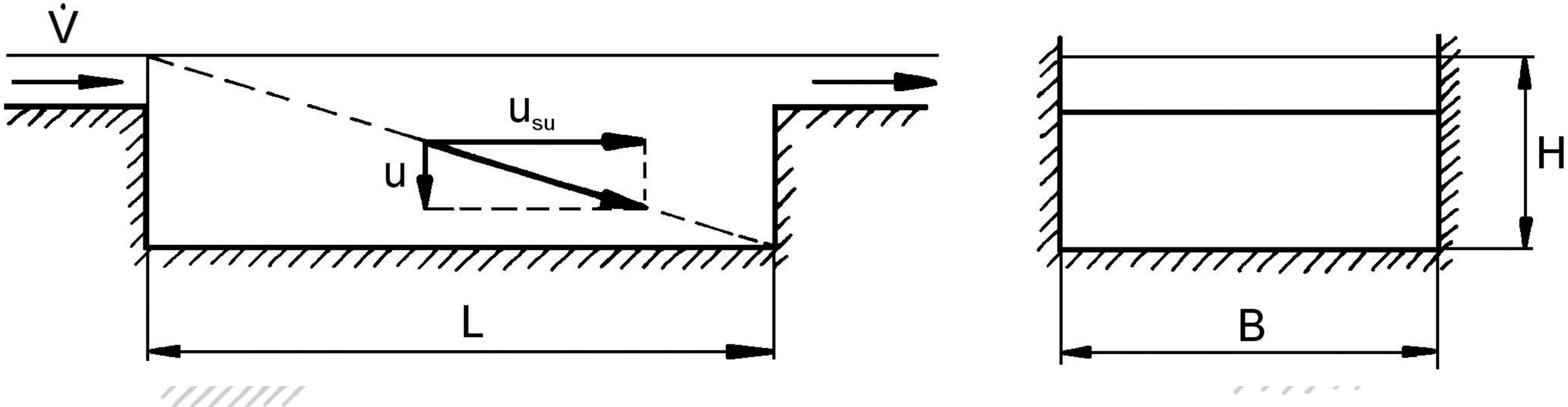
# Lamella settler




***1** – přívodní trubka, **2** – nátoková komora, **2a** – vstupní část, **2b** – výstupní část,*
***3** – horní stěny, **4** – přepadové žlaby, **5** – štěrbinové rozvaděče, **6** – lamely,*
***7** – účinný prostor, **8** – zahušťovací prostor, **9** – míchací a vyhrnovací šnek,*
***10** – spodní nádoba, **11** – odtahová trubka*

# EXAMPLE: Rectangular settler – sand trap

Design basic dimensions of gravity rectangular settler (sand trap) for separation sand particles from water. Required volumetric capacity of settler is 25 $m^3 \cdot min^{-1}$. Size of sand particles was determined by sedimentation test (by settling velocity) in solution of glycerin ($\rho$ = 1226 $kg \cdot m^{-3}$ and $\mu$ = 133 mPa·s). The smallest sand grain travels in measuring cylinder trajectory 250 mm under 100 s.



# Hydraulic classification

Separation of materials by particle size and density. ***Hydraulic classification*** Uses different settling velocities of particles with different sizes or different densities. Particles settling with the same velocity.

$$u = \sqrt{\frac{4}{3}\frac{D(\rho_s - \rho)g}{C_D \rho}}$$

## *Semi-continuous classification equipment*

Settling channel divided to several sections, the smaller settling velocity, the greater trajectory of particle.

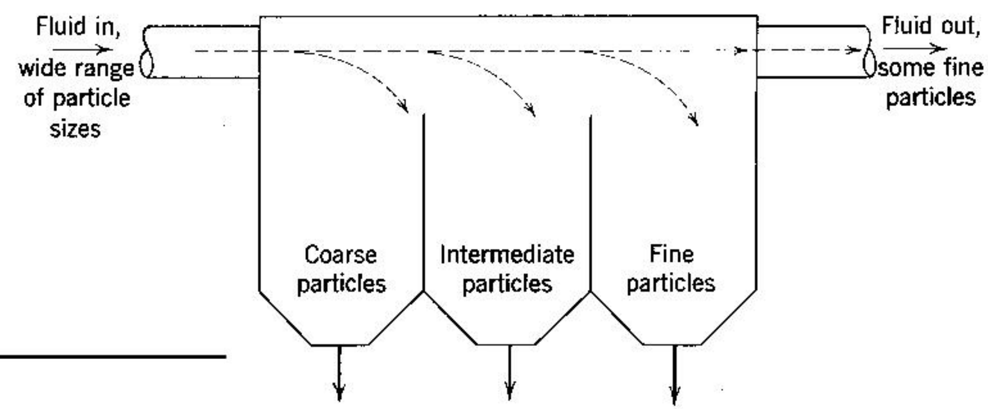
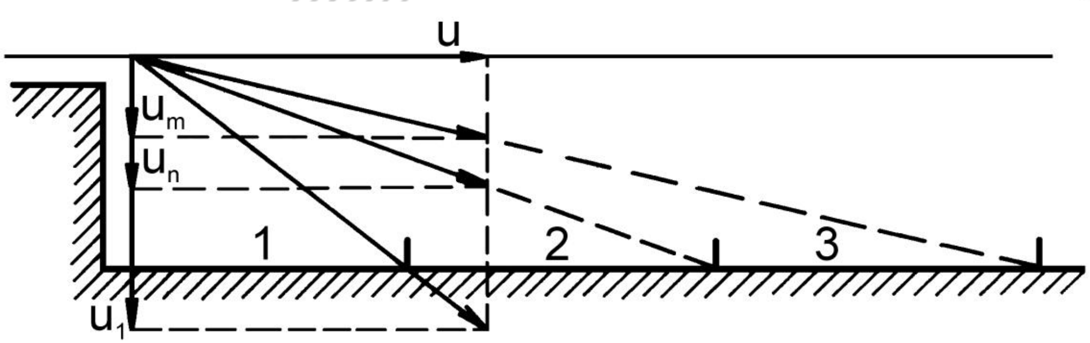


**Figure 22.4.** Gravity-settling tank.



# *Continuous classification equipment*

## *Spitzkasten*

Series of conical or pyramidal vessels of increasing diameter in the direction of flow, lowering of velocity.

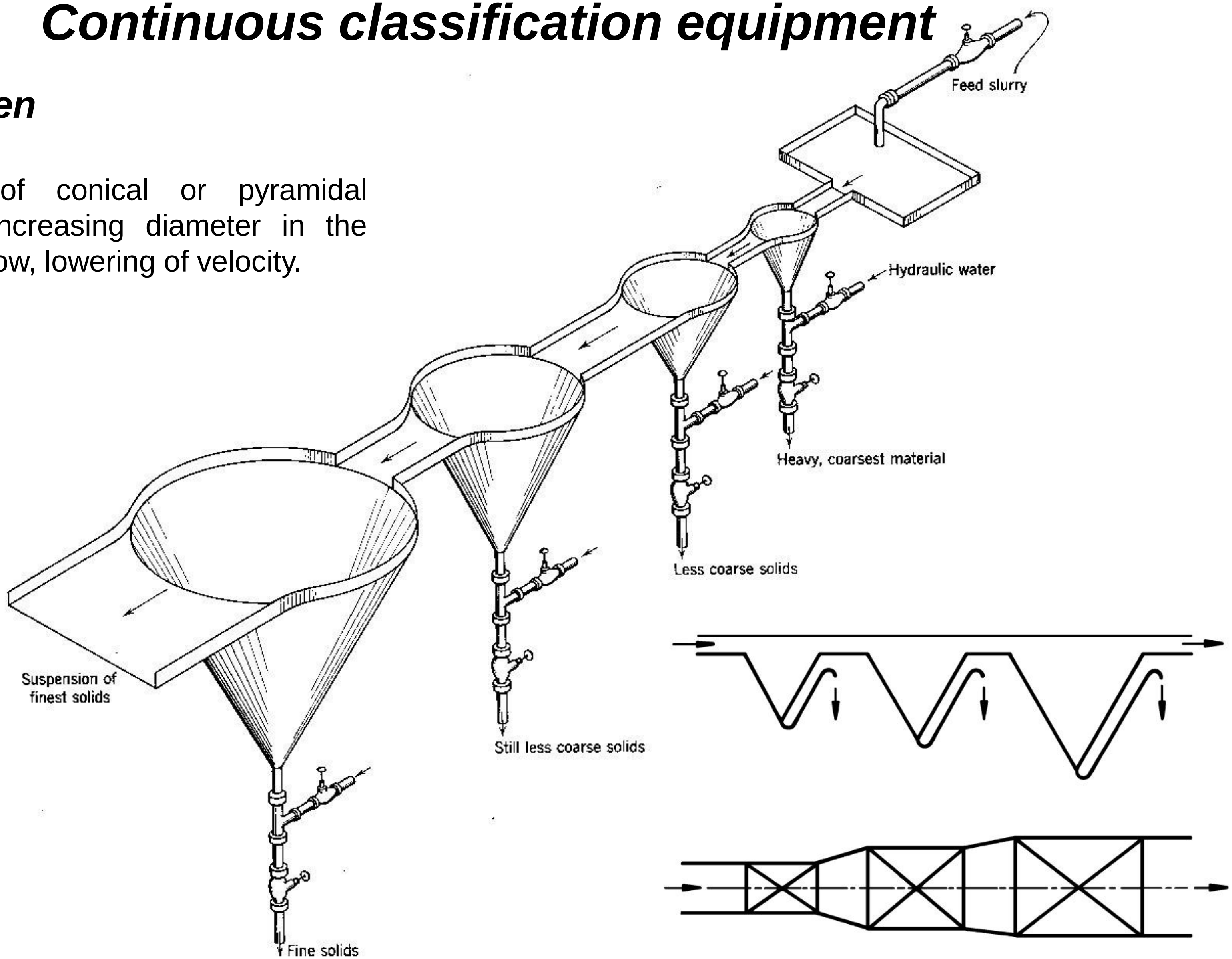
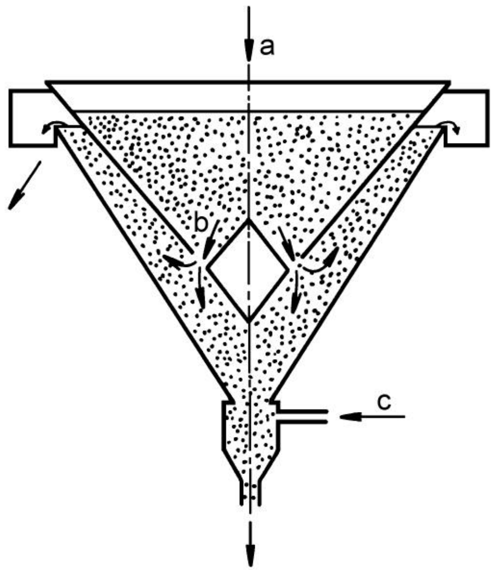



Figure 22.5. Schematic representation of a Spitzkasten.

## ***Double-cone classifier***

Change of inner movable cone position leads to change of channel cross-section.



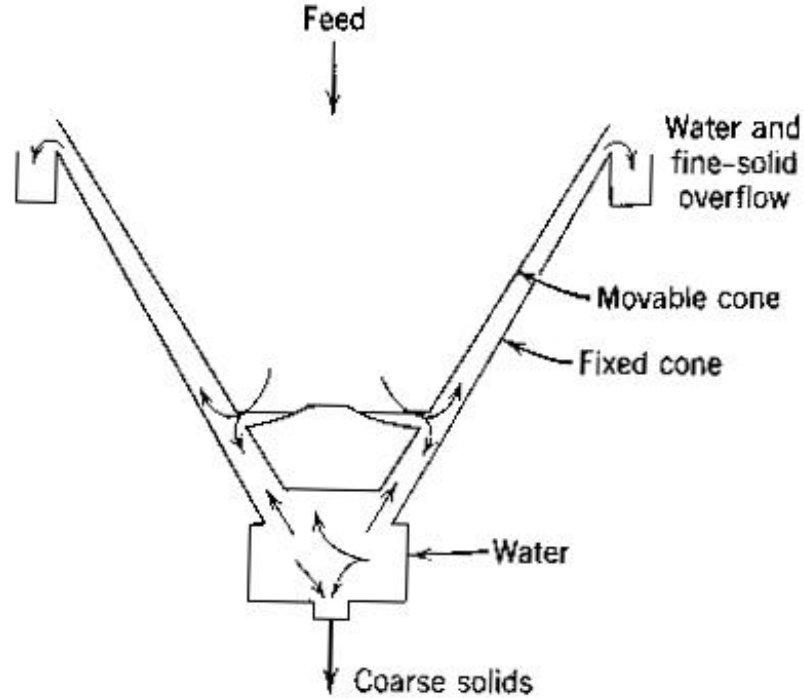




**Figure 22.7.** Double-cone classifier.

## ***Elutriators – vertical columns***

Particles which settle at a velocity higher than that of the rising fluid are collected at the bottom of column and the smaller particles are carried out of the top of the column. Several columns of different diameters in series may be used to bring about a further separation.

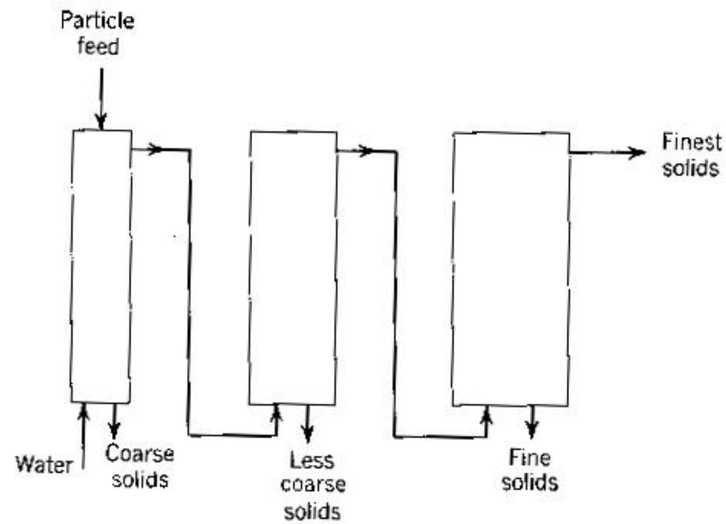
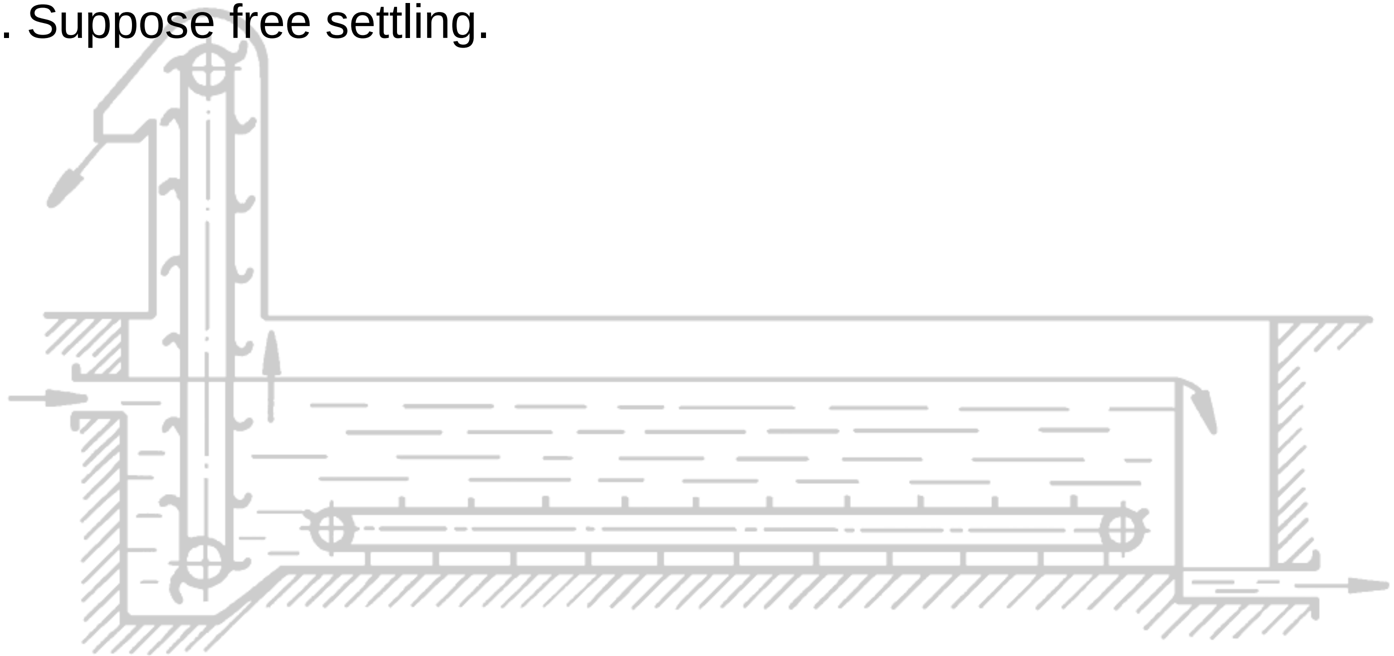



**Figure 22.6.** Elutriators.

# EXAMPLE: Hydraulic classification

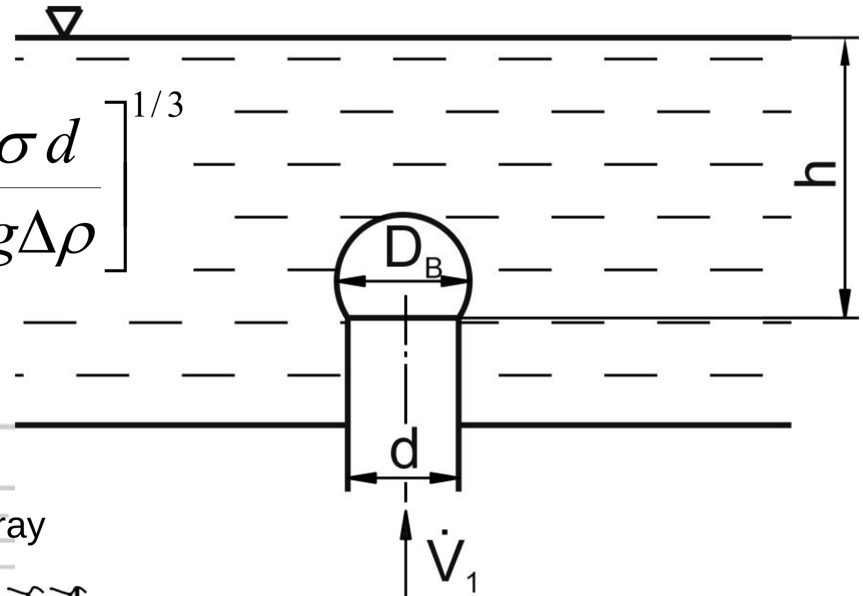
Determine water velocity ($T$ = 30 °C, $\rho$ = 998 kg·m$^{-3}$) for separaton pure galenite from mixture of siliceous ($\rho_{sk}$ = 2600 kg·m$^{-3}$) and galenic ($\rho_{sg}$ = 2600 kg·m$^{-3}$) grains. Further determine range of diameter of separated pure galenic grains. Suppose free settling.



# ***Bubbling (barbotage)***

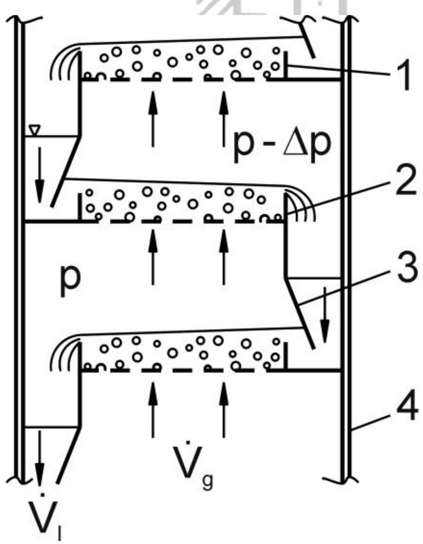
Gravity bubble movement in tray towers and flotation.

$$D_B = 1{,}82\left[\frac{\sigma\, d}{g\Delta\rho}\right]^{1/3}$$

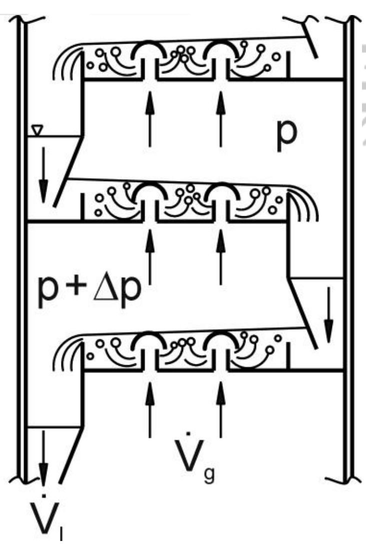



***Tray towers***

Sieve-tray

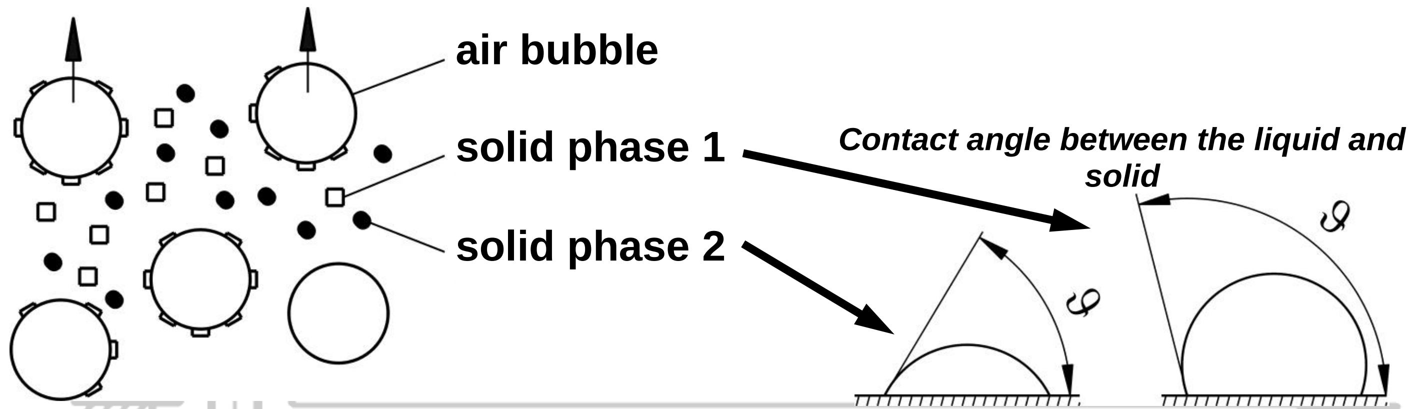


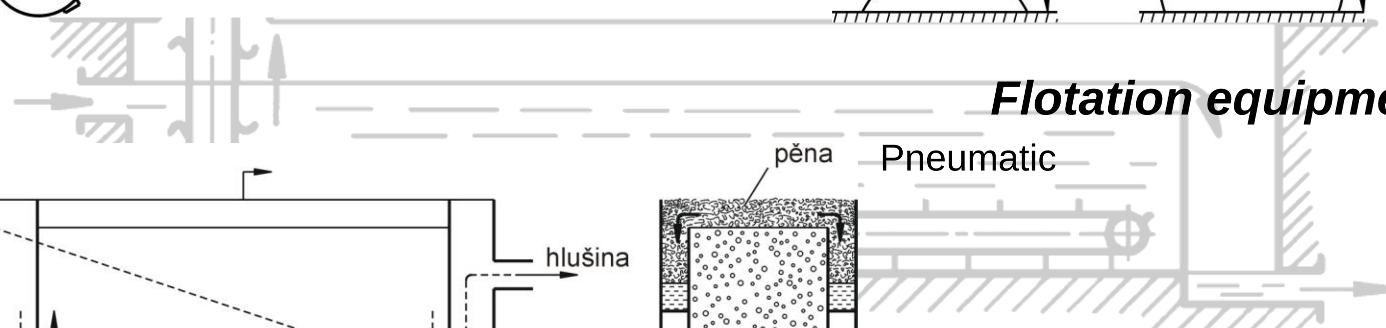

Bubble-cap tray



# Flotation

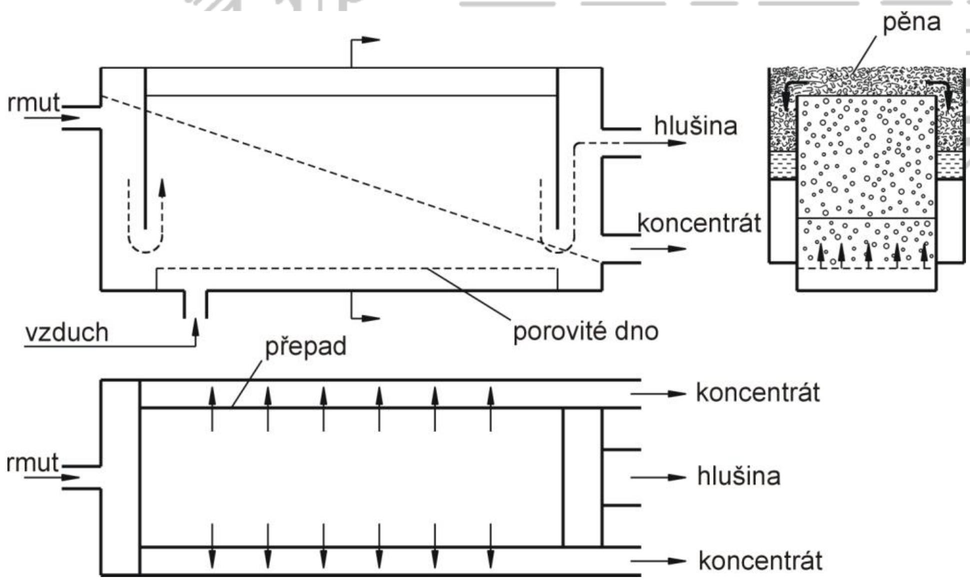
Separation of unwettable particles by rising bubbles.



## Flotation equipment

Pneumatic




Agitated