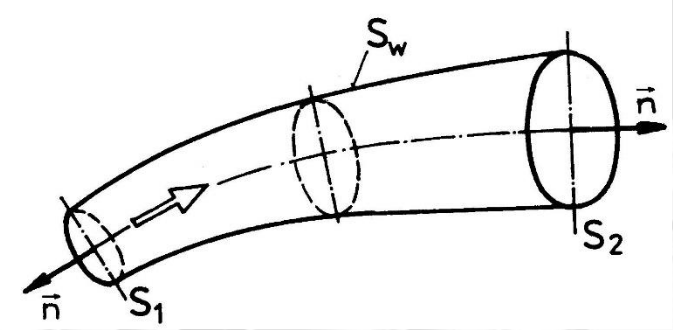
FLOW IN PIPES, PIPE NETWORKS

Continuity equation – mass balance (G54)



$$\bar{u}_1 S_1 = \bar{u}_2 S_2$$

Bernoulli equation – mechanical-energy balance (G71 – 74)

$$\kappa_1^2 \frac{\bar{u}_1^2}{2} + \frac{p_1}{\rho} + gh_1 = \kappa_2^2 \frac{\bar{u}_2^2}{2} + \frac{p_2}{\rho} + gh_2 + e_z$$

Turbulent flow: $\kappa \to 1$

Laminar flow: $\kappa^2 \frac{\bar{u}^2}{2} \to 0$ (neglectable)

mechanical-energy loss

$$e_z = \frac{p_1 - p_2}{\rho} = \frac{-\Delta p}{\rho} = \frac{\Delta p_z}{\rho}$$

Mechanical-energy loss for flow in pipe

Mechanical-energy loss due to skin friction for incompressible fluid (liquids) (G90 – 96)

$$e_z = \lambda \frac{l}{d} \frac{\overline{u}^2}{2}$$

Friction factor λ

Laminar flow: $\lambda = \dfrac{A}{Re}$ (pipe with circular cross-section A = 64)

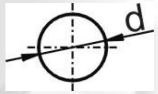
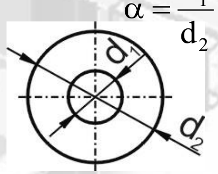
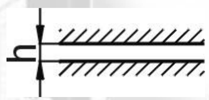
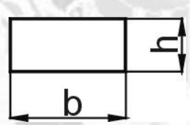
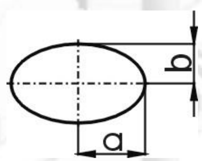
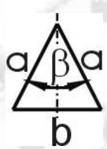
$$d_e = \frac{4S}{O} = 4\frac{cross-sectional\ area\ of\ chanel}{wetted\ perimeter\ of\ chanel}$$

Turbulent flow: $\lambda = f\left(Re, k^*\right)$ (noncircular cross-section G105)

Reynolds number $Re = \dfrac{\overline{u} d \rho}{\mu}$

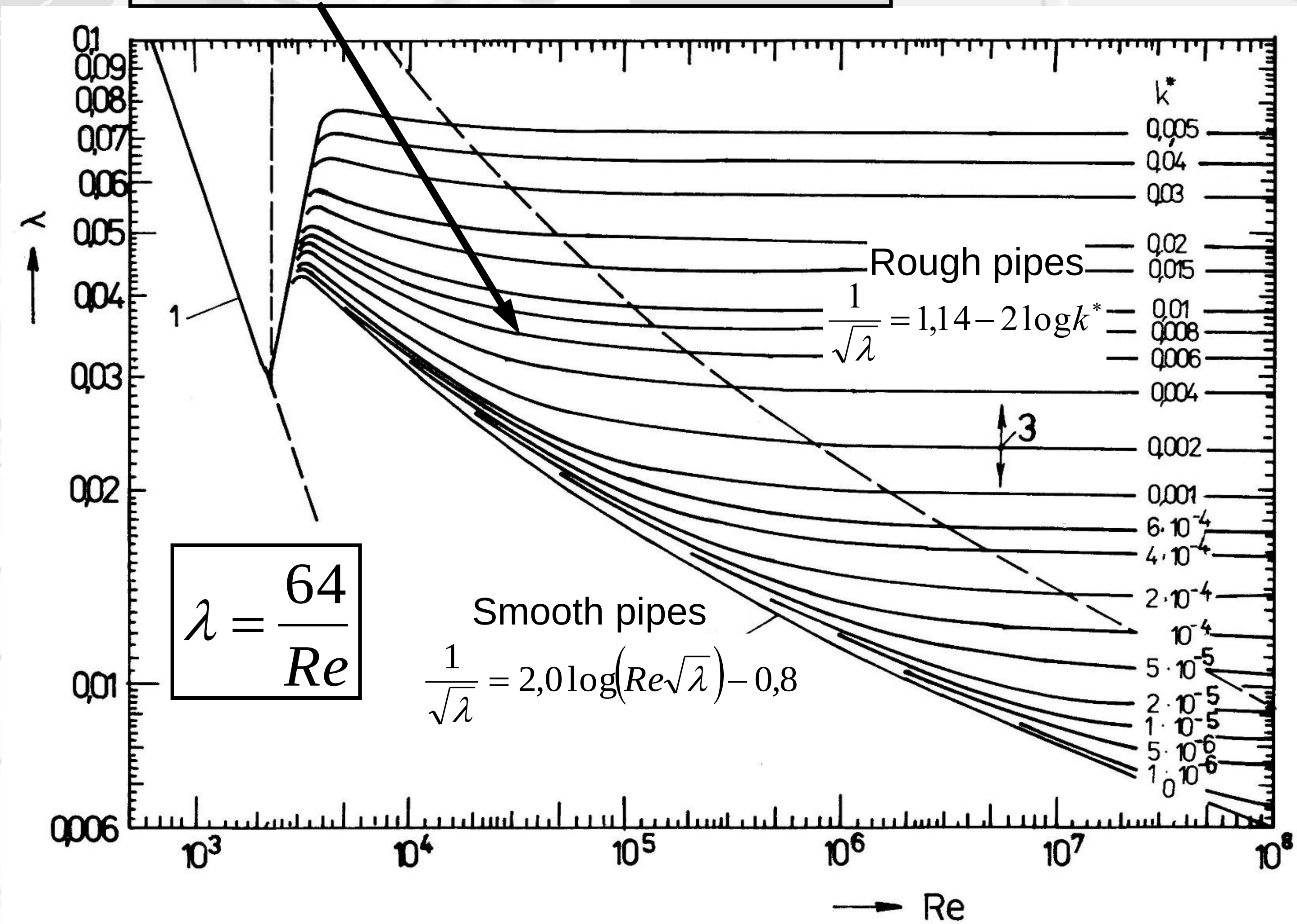
$k^* = \dfrac{k_{av}}{d}$ relative roughness of pipe wall

Values of constant A for various shapes of cross-section

Shape of cross-section		*Charact. length*	*Hydraulic diameter*	*Equation for computation of parameter A*	*A*
Circle			d	–	64
Annulus	$\alpha = 10^{-2}$ $\alpha = 10^{-1}$ $\alpha = 0{,}5$	$\alpha = \frac{d_1}{d_2}$ 	$d_2 - d_1$	$A = 64\frac{(1-\alpha)^2}{1+\alpha^2+\frac{1-\alpha^2}{\ln\alpha}}$	80,11 89,37 95,25
Slit			$2h$	–	96
Rectangle	$h/b = 10^{-2}$ $h/b = 10^{-1}$ $h/b = 1$		$\frac{2bh}{b+h}$	$A = \frac{96}{\left(1+\frac{h}{b}\right)^2}\frac{1}{\left[1-\frac{192h}{\pi^5 b}\sum_{n=1,3,5...}^{\infty}\frac{1}{n^5}\,\mathrm{tgh}\left(\frac{n\pi b}{2h}\right)\right]}$	94,71 84,68 56,91
Ellipse	$b/a = 0{,}1$ $b/a = 0{,}25$ $b/a = 0{,}5$		$\frac{4ab}{a+b}$	$A = \frac{128\left[1+\left(\frac{b}{a}\right)^2\right]}{\left(1+\frac{b}{a}\right)^2}$	106,84 87,04 71,11
Isosceles triangle	$\beta = 60°$ $a = b$ $\beta = 90°$		$\frac{a\sin\beta}{1+\sin\frac{\beta}{2}}$	$A = \frac{48\left[1+\mathrm{tg}^2\left(\frac{\beta}{2}\right)\right](B+2)}{(B-2)\left[\mathrm{tg}\frac{\beta}{2}+\sqrt{1+\mathrm{tg}^2\left(\frac{\beta}{2}\right)}\right]^2}$ *kde* $B = \sqrt{4+\frac{5}{2}\left(\frac{1}{\mathrm{tg}^2(\beta/2)}-1\right)}$	53,33 52,71

*Dependence of friction factor λ on Reynolds number and relative roughness of pipe k**

$$\lambda = \left\{ 2\log\left[0{,}27k^{*} + \left(\frac{7}{Re} \right)^{0{,}9} \right] \right\}^{-2}$$



Values of absolute roughness k_{av} of pipes from different materials

Type resp. material of pipe	***k_{av} [mm]***
glass, brass, copper, drawn tubing	0,0015 ÷ 0,0025
seamless, steel drawn tubes, new	0,03 ÷ 0,06
steel welded tubes, new	0,04 ÷ 0,1
steel tubes, slightly corroded	0,15 ÷ 0,4
steel tubes, corroded	0,5 ÷ 1,5
steel tubes, galvanized	0,1 ÷ 0,15
cast iron, new	0,2 ÷ 0,6
cast iron, corroded	1 ÷ 1,5
cast iron asphalt dipped	0,1 ÷ 0,15
PVC	0,002
concrete, smooth	0,3 ÷ 0,8
concrete, rough	1 ÷ 3
asbestos cement tubes	0,03 ÷ 0,1

EXAMPLE: *Friction loss for flow in pipe*

56 l•s^{-1} of liquid with temperature 25°C flow in horizontal slightly corroded steel tubes with length 600 m with inside diameter d = 150 mm. Determine value of pressure drop and loss due to skin friction in pipe.

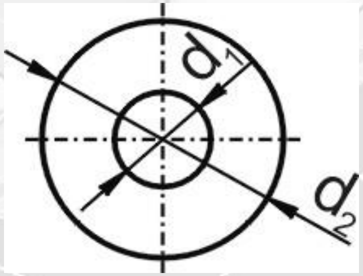
Liquid:

a) water

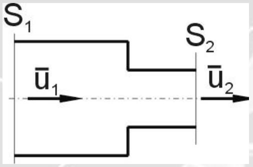
b) 98 % aqueous solution of glycerol (ρ = 1255 kg•m^{-3}, μ = 629 mPa•s)

EXAMPLE: *Friction loss for flow in pipe with noncircular cross-section*

Determine value of pressure drop in heat exchanger pipe in pipe with annulus cros-section. 98 % aqueous solution of glycerol with temperature 25°C (ρ = 1255 kg•m^{-3}, □μ = 629 mPa•s) has mass flow rate 40 kg•min^{-1}. Outside diameter of inside tube is d_1 = 32 mm and inside diameter of outside tube is d_2 = 51 mm. Length of exchanger is L = 25 m.

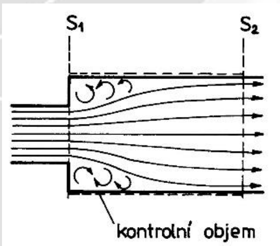


Friction losses in expansion, contraction, pipe fittings and valves (G98-102)



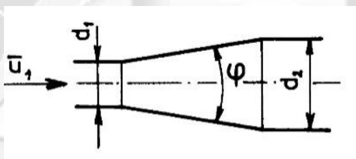
loss coefficient ζ

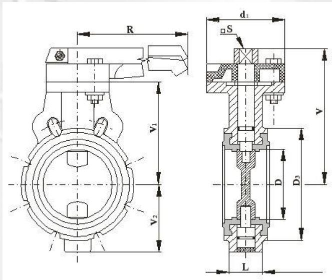
$$e_z = \zeta \frac{\bar{u}^2}{2}$$

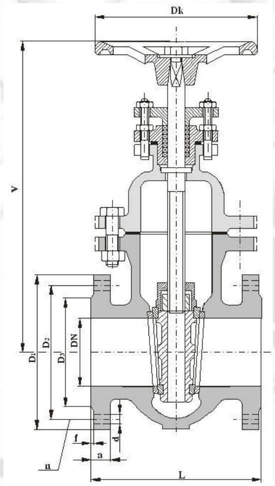


$$e_z = \lambda \frac{l_e}{d} \frac{\bar{u}^2}{2}$$

$$l_e = \frac{\zeta}{\lambda} d$$







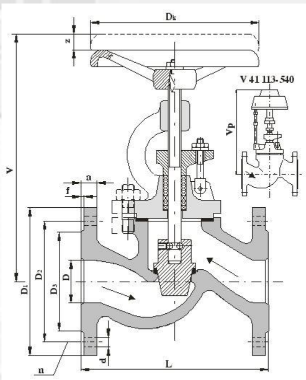
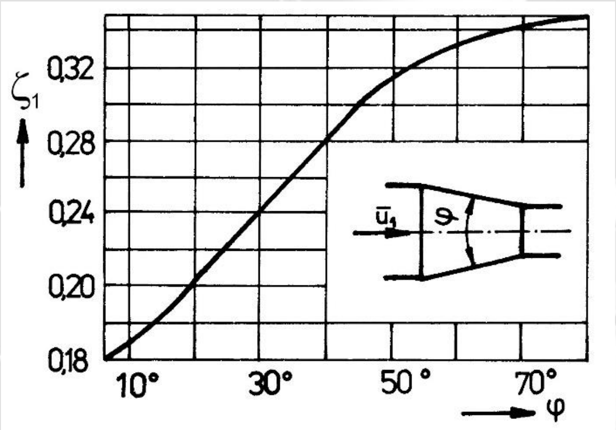


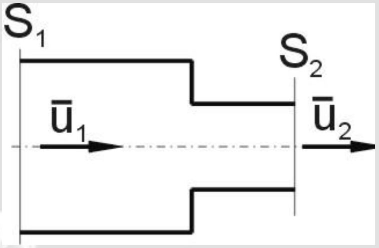
TABLE 2.10-1. *Friction Loss for Turbulent Flow Through Valves and Fittings*

Type of Fitting or Valve	*Frictional Loss, Number of Velocity Heads,* K_f	*Frictional Loss, Equivalent Length of Straight Pipe in Pipe Diameters,* L_e/D
Elbow, 45°	0.35	17
Elbow, 90°	0.75	35
Tee	1	50
Return bend	1.5	75
Coupling	0.04	2
Union	0.04	2
Gate valve		
Wide open	0.17	9
Half open	4.5	225
Globe valve		
Wide open	6.0	300
Half open	9.5	475
Angle valve, wide open	2.0	100
Check valve		
Ball	70.0	3500
Swing	2.0	100
Water meter, disk	7.0	350

Source: R. H. Perry and C. H. Chilton, *Chemical Engineers' Handbook,* 5th ed. New York: McGraw-Hill Book Company, 1973. With permission.

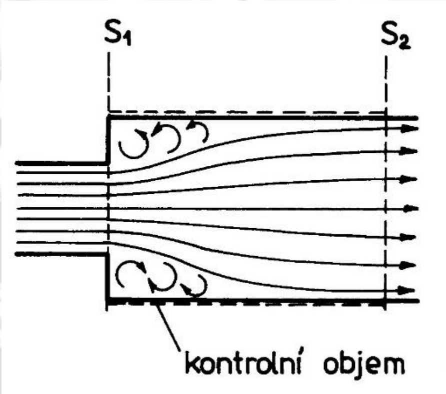
Contraction





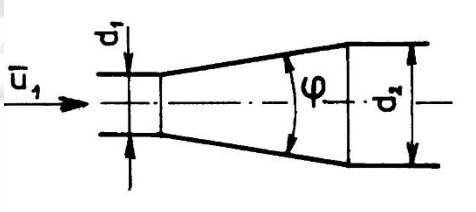
$$\zeta_2 = 0{,}5\left(1-\frac{S_2}{S_1}\right)$$

Expansion

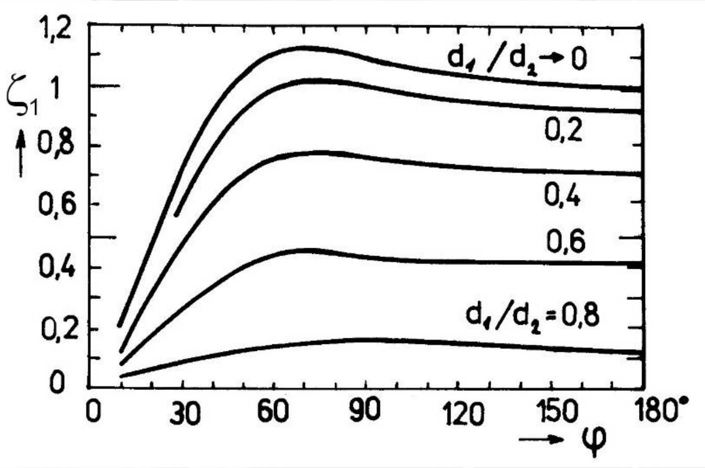


$$\zeta_1 = \left(1-\frac{S_1}{S_2}\right)^2$$

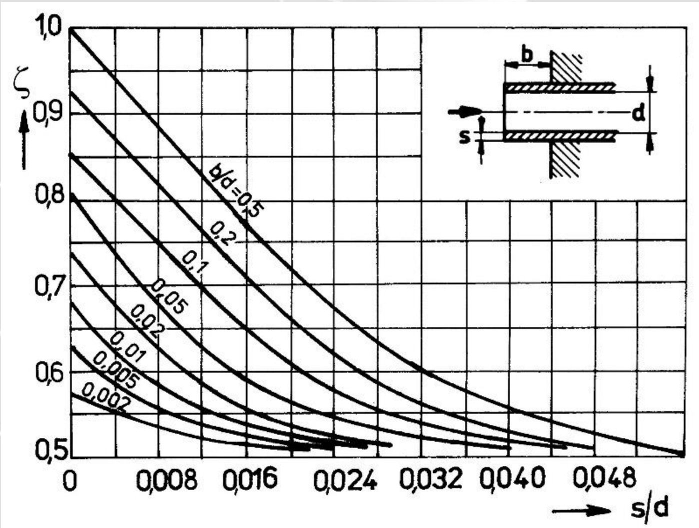
Gradual expansion (diffuser)

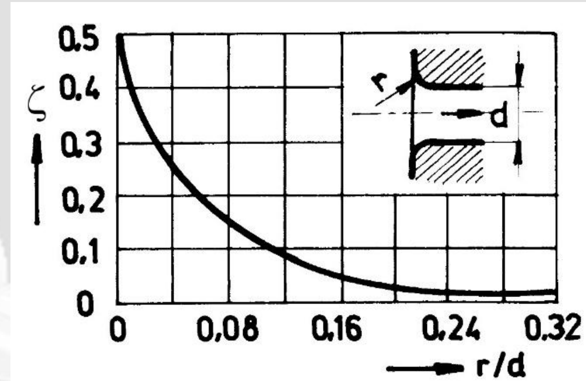


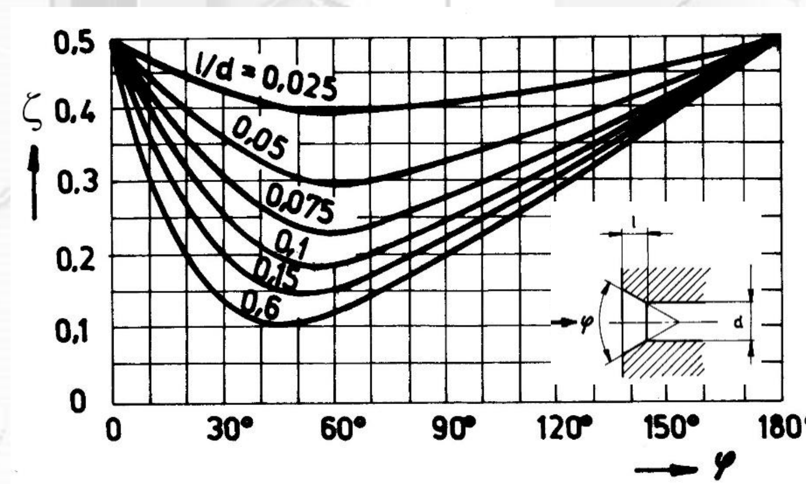
$$0<\varphi<40^\circ \;\Rightarrow\; \zeta_1=\zeta_r+\zeta_t$$



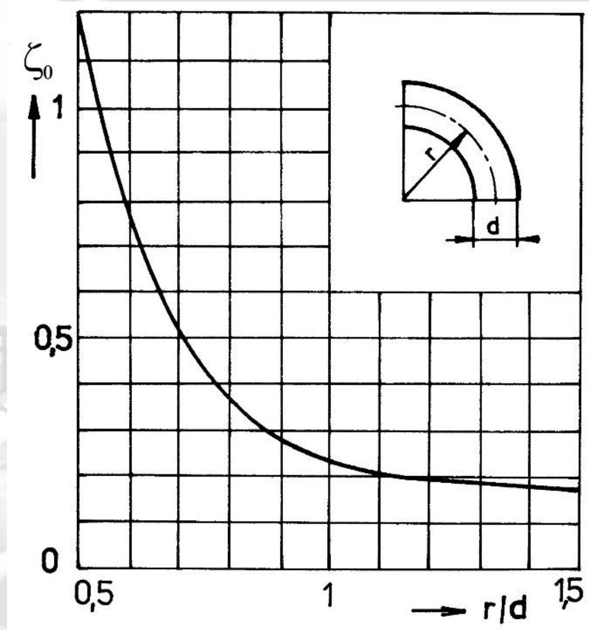
Pipe entrance







Elbow



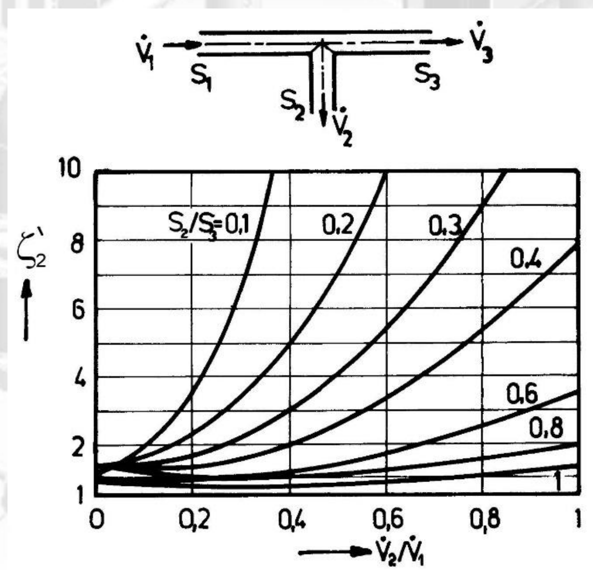
$$\zeta = \zeta_o + \zeta_t$$

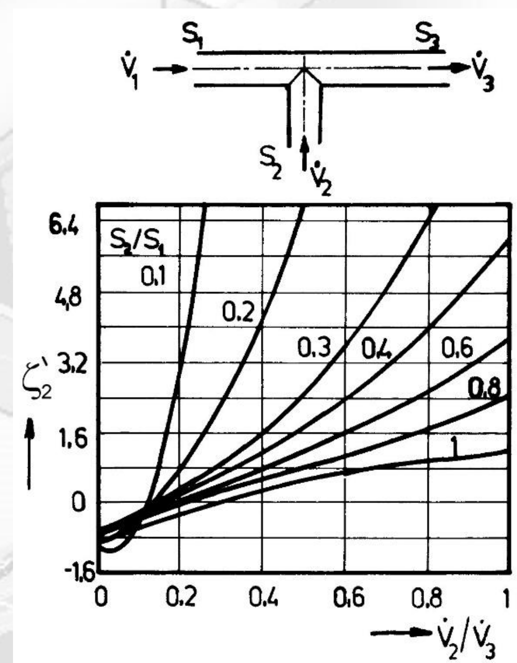
$$\zeta_t = \lambda \frac{l}{d} = \frac{\pi}{2} \lambda \frac{r}{d}$$

$$\zeta_o = 0{,}21 \left(\frac{r}{d} \right)^{-1/2}$$

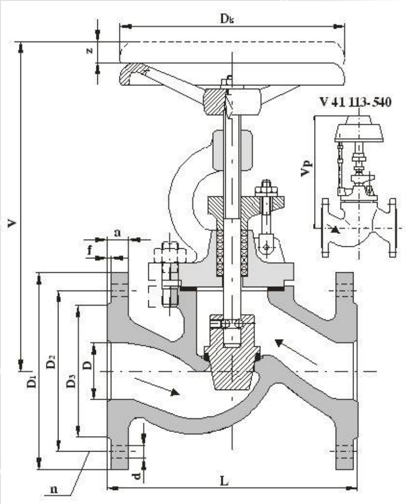
Tee

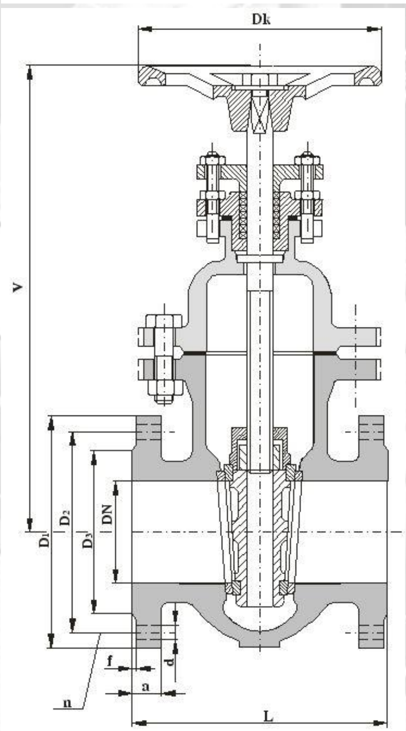
$$\Delta p = \zeta' \frac{\bar{u}^2}{2} \rho$$

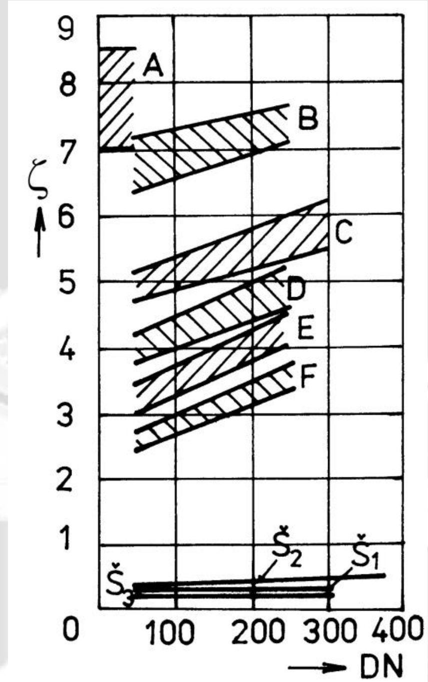




Valves







A – Check valve, screwed

B – Back straight-way valve

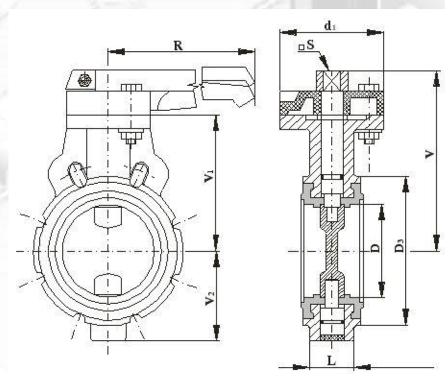
C – Check valve, casted

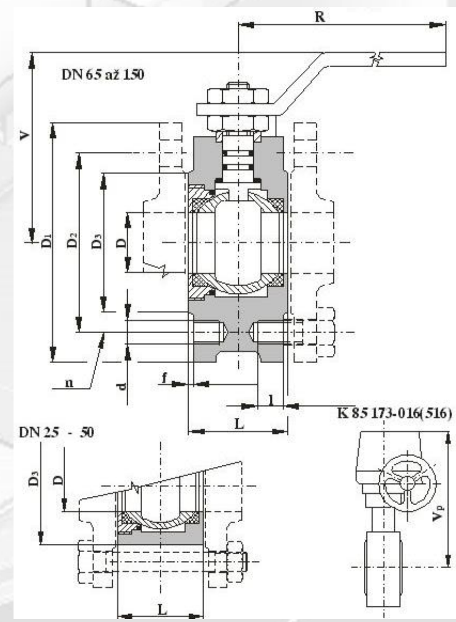
D – Back angle valve

E – Check angle valve

F – Check oblique valve

Š – Gate valves

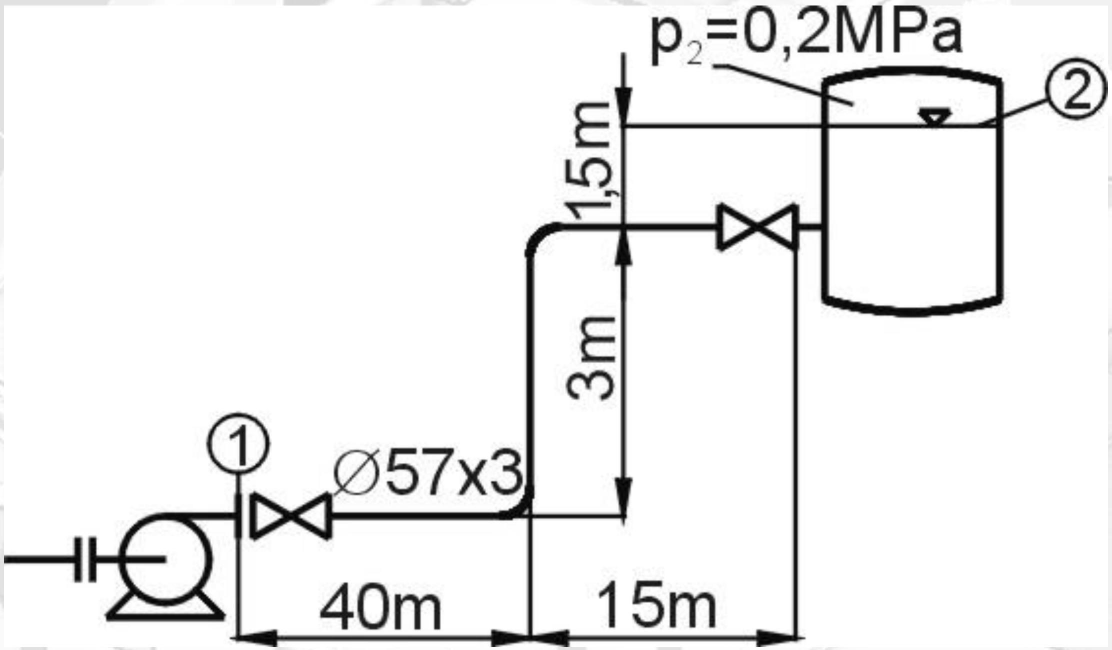




http://www.jmahod.cz

EXAMPLE: Determination of pump head pressure

Determinate head pressure of pump which give flow rate 240 l·min^{-1} of water with temperature 15 °C. Water is pumping up to storage tank with pressure over liquid surface 0.2 MPa. Pipes are made from slightly corroded steel tubes with outside diameter 57 mm and thickness of wall 3 mm.



Basic cases for pipe design

Calculation of pipe diameter at given flow rate without demand of loss (the most frequently case G107)

$$S = \frac{\dot{V}}{\bar{u}} \qquad d = \sqrt{\frac{4S}{\pi}}$$

TABLE 2.10-3. *Representative Ranges of Velocities in Steel Pipes*

Type of Fluid	*Type of Flow*	*Velocity* ft/s	*Velocity* m/s
Nonviscous liquid	Inlet to pump	2–3	0.6–0.9
	Process line or pump discharge	5.7	1.7
Viscous liquid	Inlet to pump	0.2–0.8	0.06–0.25
	Process line or pump discharge	3	0.9
Gas air	Process line	53	16
Steam 100 psig	Process line	38	11.6

Geankopolis, C. J.: ***Transport Processes and Separation Process Principles.*** 4th edition. New Jersey: Publishing as Prentice Hall PTR, 2003.1026 p. ISBN 0-13-101367-X.

Calculation of flow rate at given loss and pipe diameter

Given: mechanical-energy loss $\boldsymbol{e_z}$
dimensions of pipe ($\boldsymbol{l, d, k_{av}}$)
liquid density $\boldsymbol{\rho}$ and viscosity $\boldsymbol{\mu}$

$$e_z = \lambda \frac{l}{d}\frac{\bar{u}^2}{2} \Rightarrow \lambda = \frac{2e_z d}{\bar{u}^2 l}$$

$$\lambda = f\left(Re, k^*\right), \quad Re = \frac{\bar{u} d \rho}{\mu}$$

$$\lambda Re^2 = \frac{2e_z d}{\bar{u}^2 l}\frac{\bar{u}^2 d^2 \rho^2}{\mu^2} = \frac{2e_z \rho^2 d^3}{\mu^2 l} \Leftarrow$$

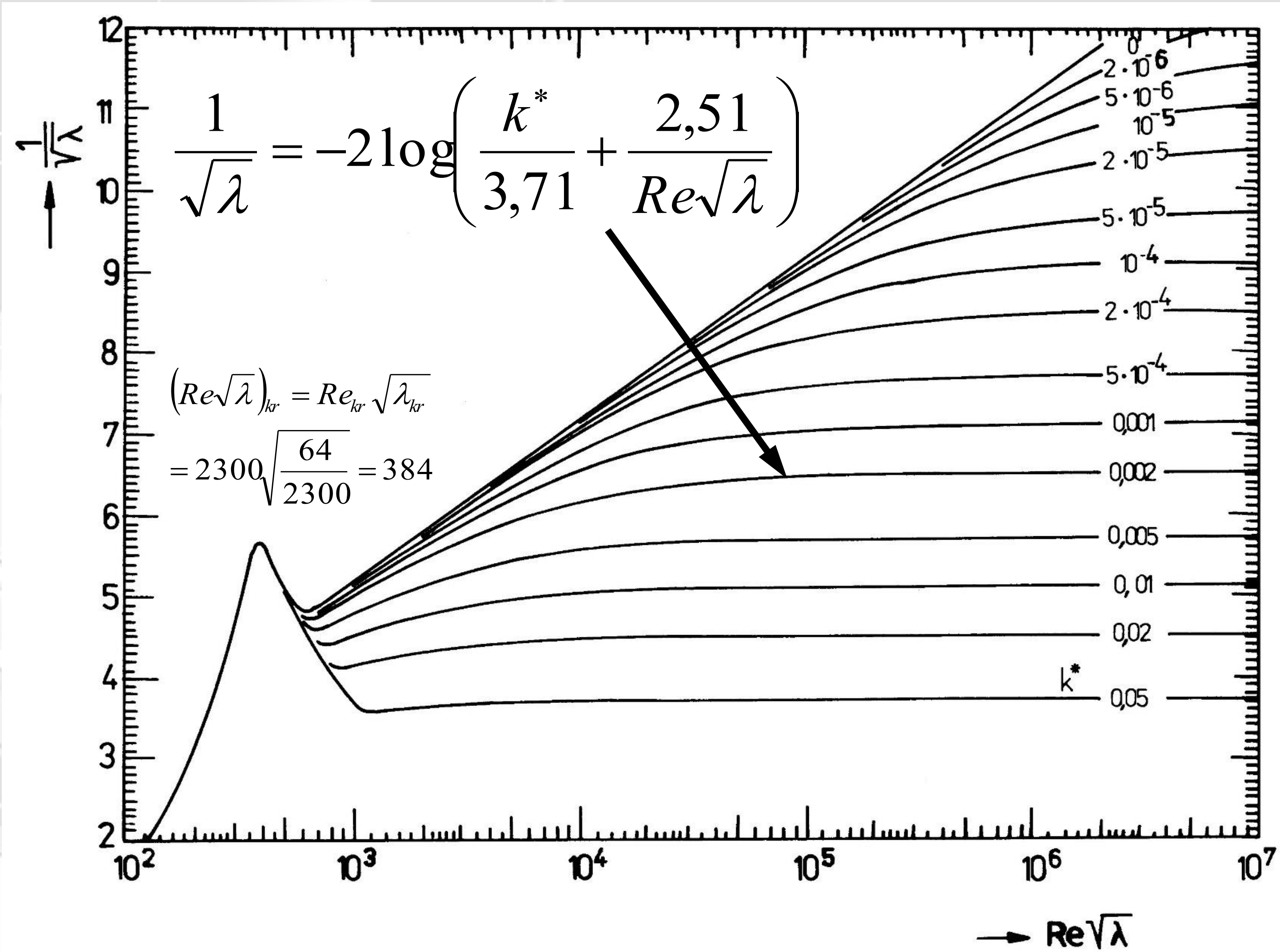
$$\Downarrow$$

$$Re\sqrt{\lambda} = \frac{d}{\mu}\sqrt{\frac{2e_z \rho^2 d}{l}}$$

$$\frac{1}{\sqrt{\lambda}} = f\left(Re\sqrt{\lambda}, k^*\right)$$

$$Re\sqrt{\lambda}\frac{1}{\sqrt{\lambda}} = Re \Rightarrow \bar{u} = \frac{\mu Re}{d\rho}$$

$$\frac{1}{\sqrt{\lambda}} = f\left(Re\sqrt{\lambda}, k^*\right)$$



$$\frac{1}{\sqrt{\lambda}} = -2\log\left(\frac{k^*}{3{,}71} + \frac{2{,}51}{Re\sqrt{\lambda}}\right)$$

$$\left(Re\sqrt{\lambda}\right)_{kr} = Re_{kr}\sqrt{\lambda_{kr}} = 2300\sqrt{\frac{64}{2300}} = 384$$

Calculation of pipe diameter at given loss and flow rate

Given: flow rate $\dot{V}$ $\qquad \bar{u} = \frac{4\dot{V}}{\pi d^2}$

mechanical-energy loss $\boldsymbol{e_z}$

pipe length $\boldsymbol{l}$

liguid density $\boldsymbol{\rho}$ and viscosity $\boldsymbol{\mu}$

$$\lambda = \frac{2e_z d}{\bar{u}^2 l} = \frac{\pi^2 e_z d^5}{8\dot{V}^2 l}$$

$$\lambda = f\left(Re, k^*\right),\ k^* = \frac{k_{stř}}{d}$$

$$Re = \frac{\bar{u} d \rho}{\mu} = \frac{4\dot{V}\rho}{\pi d \mu}$$

$$Re\sqrt[5]{\lambda} = \frac{\rho}{\mu}\sqrt[5]{\frac{128\dot{V}^3 e_z}{\pi^3 l}} \quad \Leftarrow$$

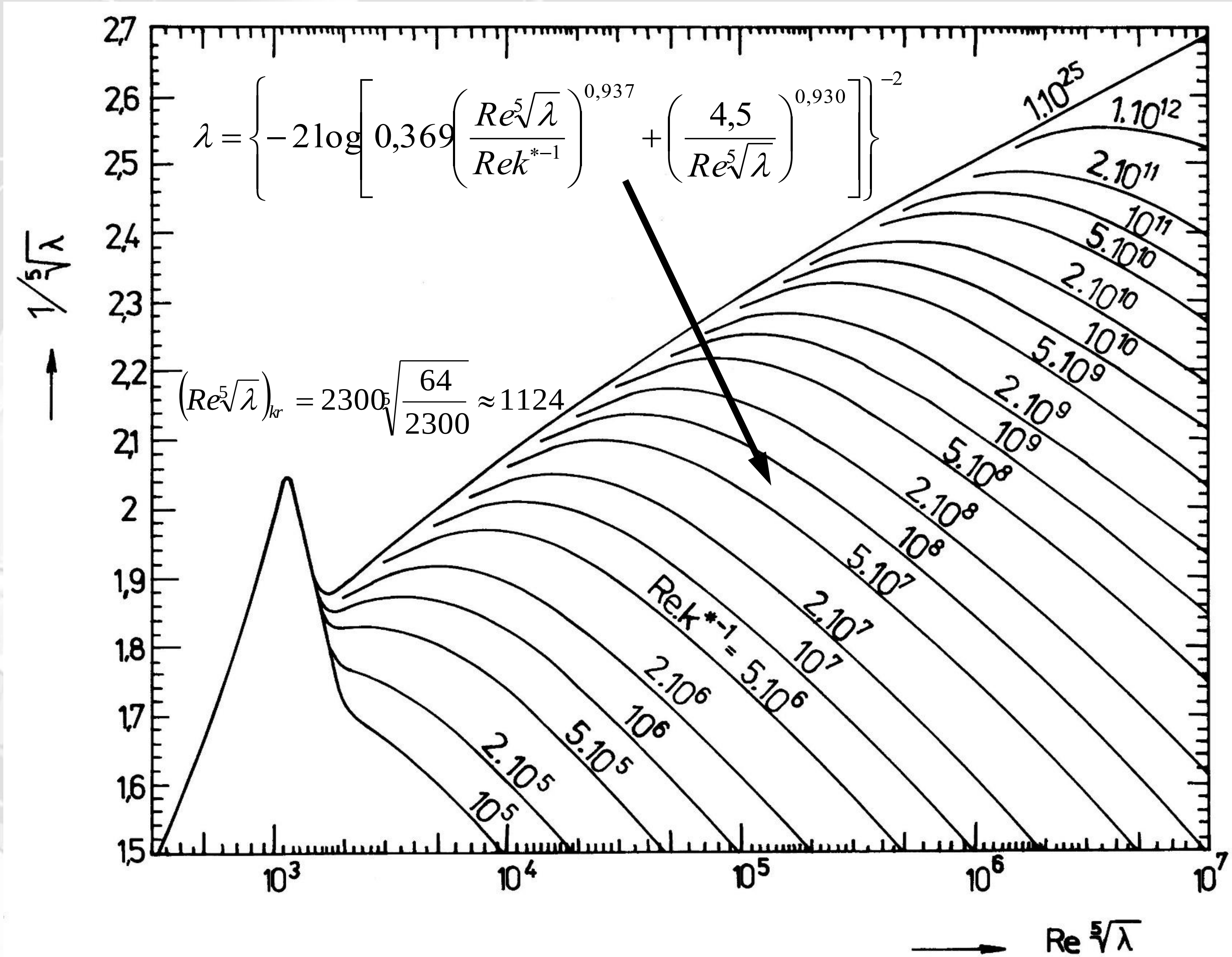
$$\Downarrow$$

$$\frac{Re}{k^*} = \frac{4\dot{V}\rho}{\pi k_{stř}\mu}$$

$$1/\sqrt[5]{\lambda} = f\left(Re\sqrt[5]{\lambda}, Rek^{*-1}\right)$$

$$Re\sqrt[5]{\lambda} \cdot \frac{1}{\sqrt[5]{\lambda}} = Re \Rightarrow d = \frac{\mu Re}{\bar{u}\rho}$$

$$1/\sqrt[5]{\lambda} = f\left(Re\sqrt[5]{\lambda}, Rek^{*-1}\right)$$



EXAMPLE: Calculation of flow rate

84 % aqueous solution of glycerol (ρ = 1220 kg•m^{-3}, μ = 99.6 mPa•s) is in tank with height of liquid surface over bases 11 m. Glycerol gravity outflow to second tank with height of liquid surface over same bases 1 m. Pipe is made from steel with outside diameter 28 mm and thickness of wall 1.5 mm and its length is 112 m. Determine volumetric flow rate of glycerol. Losses of fittings and valves are neglectable.

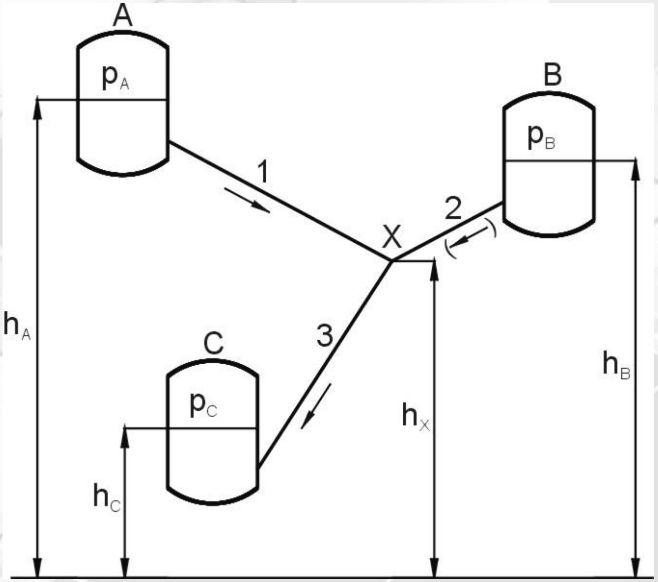
EXAMPLE: Calculation of pipe diameter

Solution of ETHANOL (ρ = 970 kg•m^{-3}, μ = 2,18 mPa•s) gravity outflow from open tank with flow rate 20 m^3•h^{-1} via pipe with length 300 m to second open tank. Liquid surface in upper tank is 2.4 m over liquid surface of second tank. Which pipe diameter is necessary for required flow rate. Pipe is made from steel with average roughness 0.2 mm. Losses of fittings and valves are express as 10 % from pipe length.

Design of pipe networks

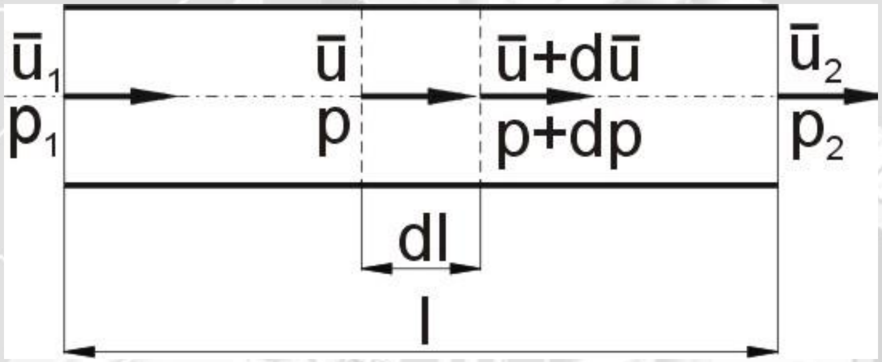
Procedure of solving:

1) Bernoulli equation for all pipes
2) Continuity equation for all nodes
3) Solve system of equations



Compressible flow of gases

Isothermal compressible flow (G107-110)



Velocity of compression wave

(velocity of sound in fluid)

$$\bar{c}^2 = \frac{p}{\rho} = pv$$

Bernoulli equation

$$\frac{1}{2}u^2 + \frac{p}{\rho} = \frac{1}{2}\left(u + \mathrm{d}u\right)^2 + \frac{p + \mathrm{d}p}{\rho} + \mathrm{d}e_z$$

$$\frac{1}{2}\bar{u}^2 - \frac{1}{2}\left(\bar{u} + \mathrm{d}\bar{u}\right)^2 - \frac{\mathrm{d}p}{\rho} - \lambda\frac{\mathrm{d}l}{d}\frac{\bar{u}^2}{2} = 0 \qquad \mathrm{d}\bar{u}^2 \to 0$$

$$\bar{u}d\bar{u} + \frac{\mathrm{d}p}{\rho} + \frac{\lambda}{d}\frac{\bar{u}^2}{2}dl = 0$$

Mass velocity (density of mass flow)

$$\overline{w} = \overline{u}\rho = \mathrm{const.}$$

$$\overline{u} = \overline{w}/\rho$$

$$\mathrm{d}\overline{u} = -\overline{w}/\rho^2\,\mathrm{d}\rho$$

$$\overline{u}d\overline{u} + \frac{\mathrm{d}\,p}{\rho} + \frac{\lambda}{d}\frac{\overline{u}^2}{2}dl = 0$$

$$\Downarrow$$

$$-\frac{\overline{w}^2}{\rho^3}\mathrm{d}\,\rho + \frac{\mathrm{d}\,p}{\rho} + \frac{\lambda\,\overline{w}^2}{d\cdot 2\rho^2}\mathrm{d}l = 0$$

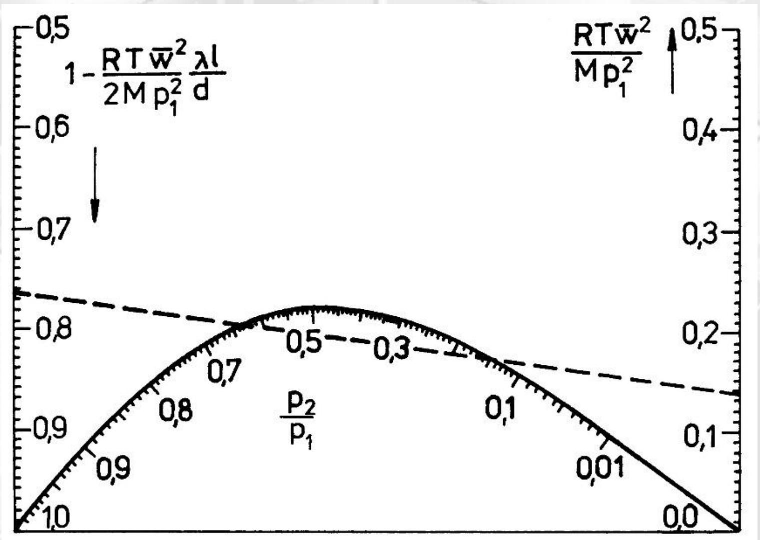
State equation for ideal gas

$$\frac{p}{\rho} = \frac{RT}{M},\ T = const. \Rightarrow \mathrm{d}\,\rho = \frac{M}{RT}\mathrm{d}\,p$$

$$\Downarrow$$

$$-\int_{p_1}^{p_2}\frac{\mathrm{d}\,p}{p} + \frac{M}{RT\overline{w}^2}\int_{p_1}^{p_2} p\,\mathrm{d}\,p + \frac{1}{2}\frac{\lambda}{d}\int_0^l \mathrm{d}l = 0$$

$$\boxed{\ln\left(\frac{p_1}{p_2}\right)^2 - \frac{M}{RT\overline{w}^2}\left(p_1^2 - p_2^2\right) + \lambda\frac{l}{d} = 0}$$



Maximum flow for compressible flow of gas

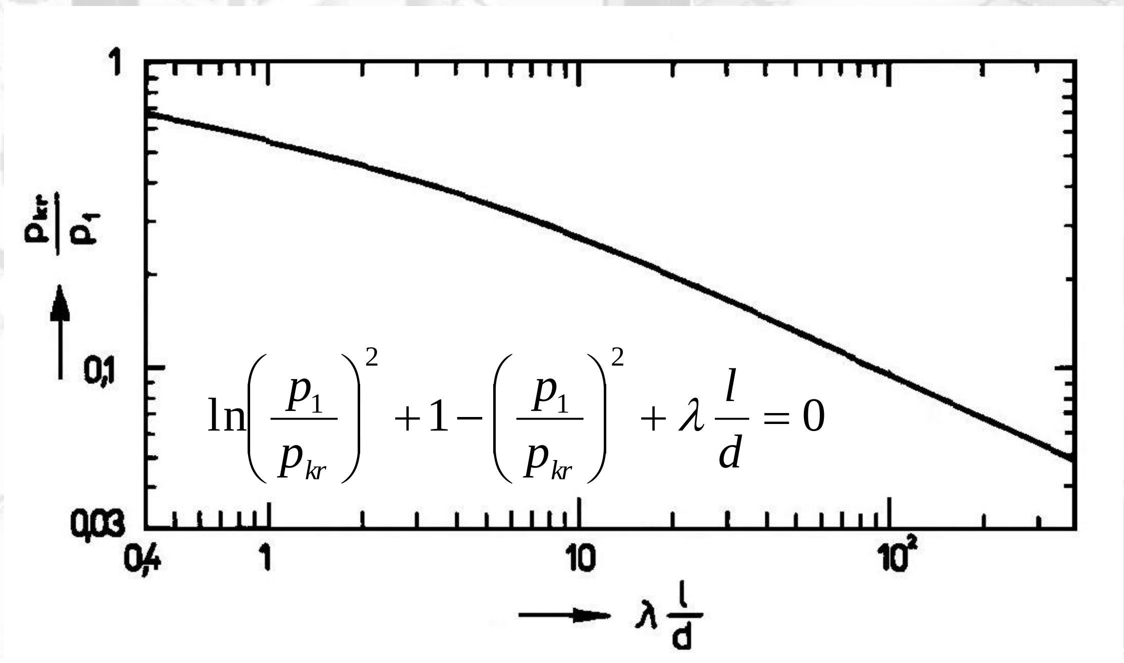
$$\mathrm{d}\,\overline{w}\,/\,\mathrm{d}\,p_2 = 0$$

$$-\frac{1}{p_2} + \frac{M}{RT\overline{w}^2}\,p_2 + \frac{M}{RT\overline{w}^3}\left(p_1^2 - p_2^2\right)\frac{\mathrm{d}\,\overline{w}}{\mathrm{d}\,p_2} = 0$$

$$\Downarrow$$

$$p_{kr}^2 = \frac{RT}{M}\,\overline{w}_{kr}^2$$

$$\overline{u}_{kr} = \frac{\overline{w}_{kr}}{\rho_{kr}} = \sqrt{p_{kr}\,v_{kr}} = \sqrt{\frac{p_{kr}}{\rho_{kr}}}$$



$$\ln\left(\frac{p_1}{p_{kr}}\right)^2 + 1 - \left(\frac{p_1}{p_{kr}}\right)^2 + \lambda\,\frac{l}{d} = 0$$

EXAMPLE: *Pressure drop for flow of Methane*

Methane flow in long-distance (3 km) pipe from storage tank withhead pressure 0.6 MPa. Pipe is made from slightly corroded steel tubes with outside diameter 630 mm and thickness of wall 5 mm. Determine pressure drop for Methane mass flow rate 40 $kg \cdot s^{-1}$. Suppose isothermal flow with temperature 20 °C (dynamic viscosity of Methane is $1{,}1 \cdot 10^{-5}$ $Pa \cdot s$).