Transient measurement of heat transfer coefficient in agitated vessel

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Abstract

Heat transfer in agitated vessels is basic and very common operation in chemical, food, pharmaceutical and other industries. Heat transfer rates depend on operational parameters, geometrical configuration of a vessel and the type of agitator. In this paper, we present results of measuring the convective heat transfer coefficient on the side of mixed liquid, based on transient heating and cooling of Newtonian liquid. A six-blade turbine impeller with pitch angle of 45 degrees was used to verify this method. Results are summarized using the Nusselt number describing its dependency on the Reynolds number and other parameters.

1 Introduction

Mixing is basic technological operation which takes place in many equipments of chemical, food, pharmaceutical and other industries. The necessity of cooling or heating the agitated liquid accompanies such processes very frequently. Usually, heat transfer is implemented by a jacket with a stream of cooling or heating medium. Plain jacket configurations, spiral, and others are used. An internal helical pipe coil or tube baffles (and many others) are often used because of lower cost and higher heat transfer coefficients.

Heat transfer rates and hence time of cooling or heating of agitated liquid is influenced by many parameters like geometry, process parameters, an impeller type and its rotation speed (mixing intensity). The heat transfer coefficient on the side of agitated liquid depends on these parameters and this paper deals with measurement of this heat transfer coefficient in a vessel with the pitched six-blade turbine.

To predict amount of heat transferred in plant vessels, it is common to build and measure heat transfer coefficients on a small geometrically similar laboratory equipment. Dimensionless parameters are commonly used to calculate heat transfer in a scaled up version of the laboratory model. In the case of forced convection, the basic dimensionless numbers are the Reynolds number, for agitated vessels defined as

$$\mathrm{Re} = \frac{N d^2 \varrho}{\mu}\,, \tag{1}$$

and the Prandtl number

$$\mathrm{Pr} = \frac{\nu}{a} = \frac{\mu c_\mathrm{P}}{\lambda}\,. \tag{2}$$

In the case of agitated vessels, other geometrical parameters like the impeller to vessel diameter ratio, height of the liquid etc. can be used in correlations for the Nusselt number which contains the heat transfer coefficient itself and it is defined as

$$\mathrm{Nu} = \frac{\alpha D}{\lambda}\,, \tag{3}$$

where D is the characteristic length (vessel diameter) and λ is the liquid thermal conductivity. A general correlation in the case of forced convection reads

$$\mathrm{Nu} = f\left(\mathrm{Re}, \mathrm{Pr}, \text{geometrical parameters}\right)\,. \tag{4}$$

In practise, the influence of temperature on the mixed liquid thermophysical properties is often represented by Sieder-Tate's correction factor, which is the last term in Eq. (5) The most common representation of the previous correlation (4) is

$$\mathrm{Nu} = c\,\mathrm{Re}^m \mathrm{Pr}^n \mathrm{Vi}^s\,, \tag{5}$$

where the Reynolds number exponent m is usually within range $2/3$ through $3/4$, the Prandtl number exponent is usually $n = 1/3$, and Sieder-Tate's correction exponent is $s = 0.14$. The viscosity ratio number Vi in Eq. (5) is defined

$$\mathrm{Vi} = \frac{\bar{\mu}}{\mu_{\mathrm{w}}}\,, \tag{6}$$

where $\bar{\mu}$ is dynamic viscosity of the mixed liquid at bulk average temperature, and μ_{w} is viscosity at the wall temperature. A little more complicated correlations are used for proximity (slow speed) impellers, for example

$$\mathrm{Nu} = c\,\left(\mathrm{Re}^m \mathrm{Pr}^n + d\right)^r \mathrm{Vi}^s\,. \tag{7}$$

A lot of correlations can be found in literature for the case of heat transfer in agitated vessels and six-blade impeller with pitched angle $45°$. Chisholm (1988) presented the following correlation

$$\mathrm{Nu} = 0.74\,\mathrm{Re}^{2/3} \mathrm{Pr}^{1/3} \mathrm{Vi}^{0.14}\,. \tag{8}$$

Similar equations were published by Paul et al. (2004), or by Edwards and Wilkinson (1972). Rieger et al. (1995) assigns this correlation to the flat six-blade turbine impeller and for the pitched six-blade impeller introduced

$$\mathrm{Nu} = 0.56\,\mathrm{Re}^{0.67} \mathrm{Pr}^{1/3} \mathrm{Vi}^{0.14}\,. \tag{9}$$

Edwards and Wilkinson (1972) introduced another correlation for the pitched six-blade turbine impeller

$$\mathrm{Nu} = 2.71\,\mathrm{Re}^{0.55} \mathrm{Pr}^{0.3}\,, \tag{10}$$

which differs significantly in constants and exponent values. Stręk and Karcz (1999) published for the pitched six-blade turbine impeller

$$\mathrm{Nu} = 0.429\,\mathrm{Re}^{0.67} \mathrm{Pr}^{0.33} \mathrm{Vi}^{0.14} \tag{11}$$

and

$$\mathrm{Nu} = 0.354\,\mathrm{Re}^{0.67} \mathrm{Pr}^{0.33} \mathrm{Vi}^{0.14}\,. \tag{12}$$

2 Theory of transient method

Heat transfer coefficient α on the side of agitated liquid describes the heat transfer rate between the vessel wall and mixed liquid. Supposing a perfectly mixed liquid, its temperature T is assumed to be uniform through out the whole liquid volume. If we know the wall temperature T_{w}, the heat flux can be calculated as

$$q = \frac{\dot{Q}}{S} = \alpha \left(T_{\mathrm{w}} - T \right) , \tag{13}$$

where $\dot{Q}$ is total heat flow rate crossing a wall surface S. This total heat flow rate can be expressed from enthalpy balance of the heating (cooling) liquid as

$$\dot{Q} = \dot{m}_{\mathrm{A}} c_{\mathrm{PA}} \left(T'_{\mathrm{A}} - T''_{\mathrm{A}} \right) , \tag{14}$$

supposing that the system is perfectly insulated from surroundings (except the inlet and outlet streams of heating liquid). The specific heat capacity c_{PA} is considered to be practically invariant here (determined at bulk liquid mean temperature).

The total heat flow rate $\dot{Q}$ at specific time point can be expressed from the unsteady enthalpy balance of the mixed liquid. Assuming a perfectly insulated system, no volume sources of heat (like dissipation of the impeller mechanical energy), no change of the liquid mass M, and negligible temperature dependency of the specific heat capacity, this balance could be written as

$$M c_{\mathrm{P}} \frac{\mathrm{d}T}{\mathrm{d}t} = \dot{Q} \, . \tag{15}$$

Substituting Eq. (13), we get an ordinary differential equation of the first order

$$M c_{\mathrm{P}} \frac{\mathrm{d}T}{\mathrm{d}t} = \alpha S \left(T_{\mathrm{w}} - T \right) . \tag{16}$$

Supposing constant temperature of the wall T_{w} and constant heat transfer coefficient α, we can derive the liquid batch temperature T dependency on time

$$\frac{T - T_{\mathrm{w}}}{T_0 - T_{\mathrm{w}}} = \mathrm{e}^{-\dfrac{\alpha S}{M c_{\mathrm{P}}} t} . \tag{17}$$

Here, T_0 represents initial condition of the liquid batch temperature

$$T \Big|_{t=0} = T_0 \, . \tag{18}$$

How to use this solution to determine the convective heat transfer coefficient α? Supposing we have measured the initial temperature and the wall temperature is kept at constant value somehow, we could measure the time course of liquid temperature. Then we could find such α coefficient so that the theoretical solution (17) would be “close” to the experimental data. A mathematical formulation could read that sum of absolute deviations (or sum of squares) of experimental data T_i from theoretical solution (17) is minimal

$$\sum_{i=1}^{n} \left| T(t_i) - T_i \right| = \min \, . \tag{19}$$

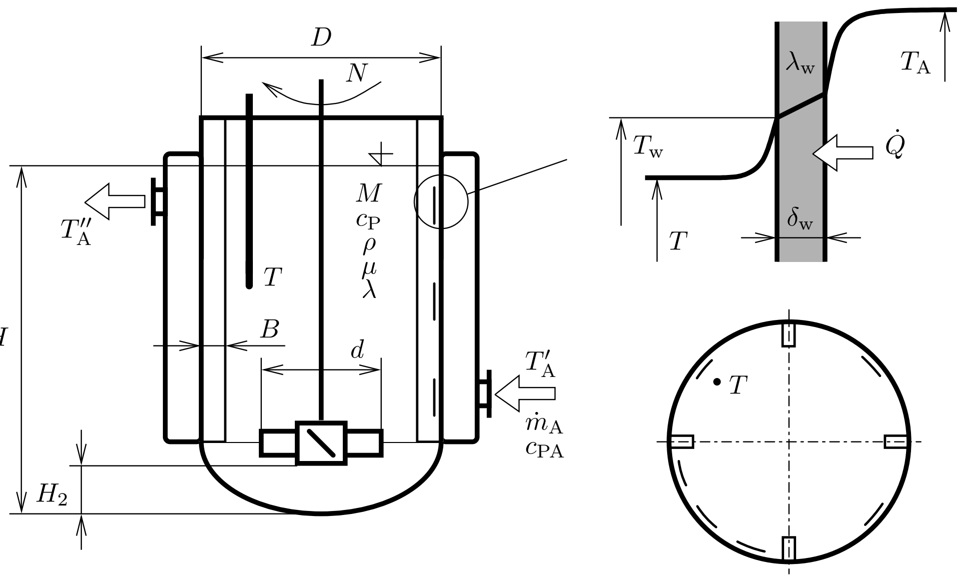

Fig. 1: Scheme of our experimental equipment.

This optimization procedure would then lead to finding an optimal value of the heat transfer coefficient α.

The solution is a little more complex in our case. Let us imagine how the real experiment usually proceeds. The liquid batch is heated up to an initial and uniform temperature T_0. The wall temperature is equal to this initial temperature as well. Then, a heating medium with inlet temperature T'_A starts flowing through the vessel jacket. The liquid batch temperature will rise and will reach the heating medium inlet temperature theoretically after infinite time. In this case, the wall temperature changes with time as well, and it should reach the same temperature as the liquid batch after infinite time.

The transient enthalpy balance (16) cannot be solved analytically in this case (because we do not know an explicit function of T_w with respect to time), but it can be solved numerically. Euler's method is the simplest one which can be used. So, in our case, the theoretical solution is an outcome of Eq. (16) numerical solution using the improved Euler method (Acheson, 1997; Petera and Dostál, 2007). This makes the optimization procedure more computationally intensive, but with today's computers it is not a problem at all - it was implemented using Matlab®.

Our case is furthermore complicated by the fact that numerical solution depends on the initial condition (18) as well as on the experimental data T_i and T_{wi}. To eliminate inaccuracy of the initial temperature measurement, it was subjected to our optimization procedure as well. So, the two-parametric optimization procedure yielded values of the heat transfer coefficient α and the initial temperature T_0.

3 Experimental

Measurements of heat transfer coefficient using the transient method described in the previous section were done in a cylindrical vessel with elliptical bottom and heated jacket situated on the cylindrical part of the vessel. Four baffles were placed along the vessel circumference by $90°$. The liquid was agitated by the six-blade turbine impeller with pitch angle $45°$. Geometri-

Vessel diameter	D	200	mm	
Liquid height	H	200	mm	$H/D = 1$
Baffle width	B	20	mm	$B/D = 0.1$
Number of baffles		4		
Heat transfer area	S	0.1	m^2	
Impeller type	six-blade turbine, pitched angle $45°$			
Impeller diameter	d	67	mm	$D/d = 3$
Impeller height above bottom	H_2	67	mm	$H_2/d = 1$, $H_2/D = 1/3$
Blade width	b	13	mm	$b/D = 0.065$, $b/d = 0.194$
Impeller rotation rate	N	$50 - 900$	min^{-1}	
Agitated liquid	distilled water			
Average temperature	$\overline{T}$	30	$°\mathrm{C}$	
Density at $\overline{T}$	ϱ	995.7	$\mathrm{kg\,m^{-3}}$	
Specific heat capacity	c_P	4178	$\mathrm{J\,kg^{-1}\,K^{-1}}$	
Thermal conductivity	λ	0.618	$\mathrm{W\,m^{-1}\,K^{-1}}$	
Dynamic viscosity	μ	0.7966×10^{-3}	$\mathrm{Pa\,s}$	
Prandtl number	Pr	5.39		
Thermal diffusivity	a	0.148×10^{-6}	$\mathrm{m^2\,s^{-1}}$	
Liquid mass	M	5.675	kg	

Table 1: Parameters of our experimental equipment.

cal parameters are depicted at Fig. 1 and they are summarized together with other experiment parameters in Table 1.

The agitator was driven by Servodyne 5000-25 power unit (Cole Parmer Instrument Co., $20 - 900\,\mathrm{min}^{-1}$). Distilled water was used in the vessel, and its temperature were measured by platinum resistance thermometer Pt100, which was placed in the area between the impeller and baffles, see Fig. 1. Wall temperatures were measured by thermocouples SA1-T-72 (Omega Engineering, Inc., Cu-Co type). The internal wall temperature was measured at several locations (see Fig. 1), and it was found that temperatures along the vessel perimeter were within accuracy limits of the thermocouples.

The platinum resistance thermometer signal was processed by data acquisition unit Agilent 34970A (Agilent Technologies), which contains a multichannel input multiplexer connected to a controlling computer. Integration type A/D converter is suitable to measure thermocouples signals. Temperature sampling period of 0.5 s was used, and the single temperature measurement period was 20 ms (time of A/D converter integration – period length of voltage supply).

Measurements were performed periodically. First, the agitated liquid was cooled down by the cooling water flowing through the jacket. Then, the water flow through the jacked was switched to hot water suddenly and the liquid temperature inside the vessel started to increase. Temperatures of liquid and wall were measured until they reached same value as water flowing through the jacket. Then, another cycle of cooling/heating followed. Typical time course of such cycle is at Fig. 2. Using this dependency, the heat transfer coefficient α was determined as described in the previous section. The heat transfer coefficient was not evaluated using the whole time course. It is obvious from Fig. 2 that the liquid batch temperature is within range $15°\mathrm{C}$ through $45°\mathrm{C}$ approximately. We used narrower temperature range $20 - 40°\mathrm{C}$, with mean temperature $30°\mathrm{C}$. The ambient temperature was $28°\mathrm{C}$ which was close to the mean temperature so it practically prevented heat exchange between the agitated liquid and surroundings and minimized measurement errors.

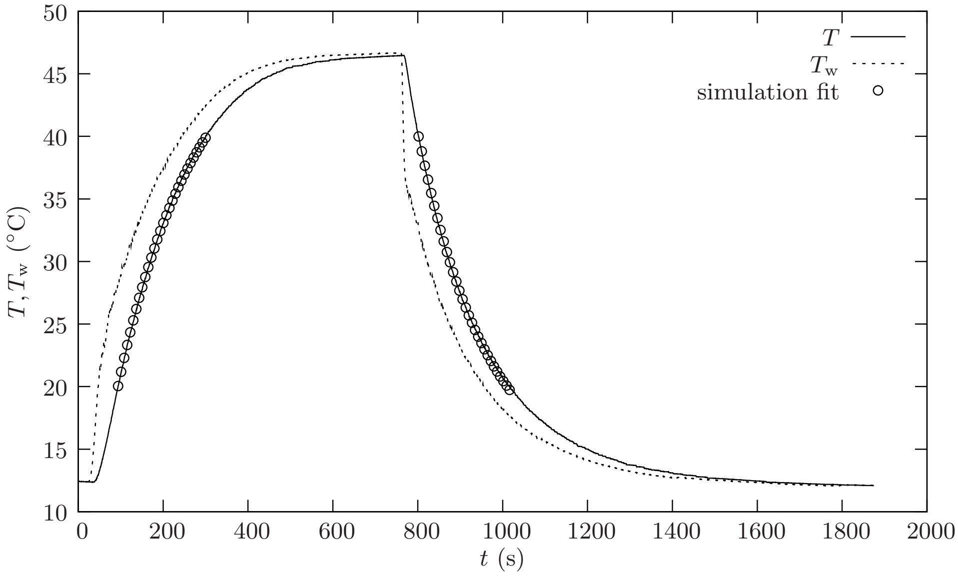

Fig. 2: Typical time course of liquid and wall temperatures during a single heating/cooling cycle, $N = 500\,\mathrm{min}^{-1}$.

$N(\mathrm{min}^{-1})$	Re	α ($\mathrm{W\,m^{-2}\,K^{-1}}$)	T_0 (°C)	$\overline{T}_w$ (°C)	Nu	$\mathrm{Vi}^{0.14}$
Heating						
50	4670	1230	19.8	38.9	398	1.0255
100	9340	1826	19.8	37.3	591	1.0211
200	18681	3106	19.8	35.6	1005	1.0161
300	28021	4213	19.6	34.8	1363	1.0138
400	37362	5180	19.9	34.7	1676	1.0136
500	46702	5753	19.8	34.3	1862	1.0126
500	46702	5956	19.8	34.3	1928	1.0125
600	56042	6796	19.9	33.7	2199	1.0108
700	65383	7390	19.9	33.2	2392	1.0095
800	74723	7998	20.0	33.5	2588	1.0102
900	84064	8655	20.0	33.1	2801	1.0090
Cooling						
50	4670	1029	39.5	20.8	333	0.9710
100	9340	1465	39.4	22.0	474	0.9751
200	18681	2823	39.4	24.0	914	0.9815
300	28021	3975	39.6	24.9	1287	0.9844
400	37362	4997	39.6	25.6	1617	0.9865
500	46702	5851	39.7	25.8	1894	0.9873
500	46702	5710	39.7	25.7	1848	0.9867
600	56042	6621	39.6	25.8	2143	0.9872
700	65383	7279	39.8	26.2	2356	0.9886
800	74723	7988	39.7	26.3	2585	0.9887
900	84064	8506	39.7	26.6	2753	0.9897

Table 2: Evaluated heat transfer coefficients and other parameters values.

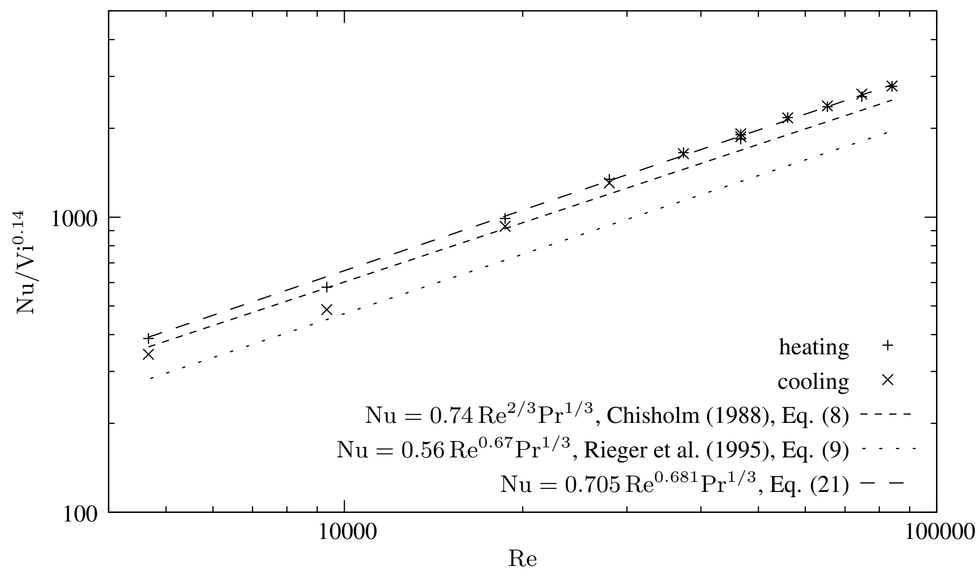

Fig. 3: Measured data and our fitted correlation (Eq. 21) compared to Chisholm (1988) and Rieger et al. (1995).

Fig. 3 shows measured values using the Nusselt number dependency on Reynolds number, compared to correlations (Chisholm, 1988; Rieger et al., 1995) suitable for the pitched six-blade turbine impeller.

4 Conclusions

Heat transfer coefficients have been measured in an agitated vessel using the transient method. Batch liquid and wall temperatures were recorded during experiments. The transient enthalpy balance was solved numerically in each step of the optimization procedure (based on minimization of absolute deviations from experimental data) to get values of heat transfer coefficients. The results agreed well with data available in literature.

Future work will concentrate on performing more experiments and estimations of measurement errors. Even with our small set of measured data, we were able to fit parameters in Eq. (5) and obtain similar values found in the literature. For example, with fixed exponent values of the Reynolds, Prandtl and Viscosity ratio numbers we get the one-parameter fit of the remaining constant c

$$\mathrm{Nu} = 0.823\,\mathrm{Re}^{2/3}\mathrm{Pr}^{1/3}\mathrm{Vi}^{0.14} \tag{20}$$

which is close to 0.74 published by Chisholm (1988). A two-parametric fit of constant c and exponent m gives

$$\mathrm{Nu} = 0.705\,\mathrm{Re}^{0.681}\mathrm{Pr}^{1/3}\mathrm{Vi}^{0.14} \tag{21}$$

Our fitting procedure was applied to all data in Table 2 except the two smallest rotation rates, 50 and $100\,\mathrm{rpm}$, where the assumption of uniform liquid temperature in the vessel is far from reality. Our heat transfer coefficients and consequently the Nusselt numbers are slightly higher than those found in literature, see Fig. 3 for graphical comparison. This is caused by the fact, that our heat transfer area is located only on the cylindrical part of the vessel where heat transfer coefficients are usually larger than at the bottom of the vessel. The literature correlations were probably obtained in situations with heated both, the cylindrical wall and the bottom of vessel jacket.

Acknowledgment

This work has been subsidized by the research project of Ministry of Education of the Czech Republic MSM6840770035.

Nomenclature

a	thermal diffusivity $(\mathrm{m^2\,s^{-1}})$
b	blade width (m)
B	baffle width (m)
c	model parameter $(-)$
c_{PA}	specific heat capacity of heating or cooling liquid A $(\mathrm{J\,kg^{-1}\,K^{-1}})$
c_{P}	specific heat capacity of agitated liquid $(\mathrm{J\,kg^{-1}\,K^{-1}})$
d	model parameter $(-)$
d	diameter of impeller (m)
D	vessel inner diameter (m)
H_2	clearance between impeller and vessel bottom (m)
H	height of agitated liquid in the vessel (m)
m	model parameter $(-)$
$\dot{m}_{\mathrm{A}}$	mass flowrate of heating or cooling liquid A $(\mathrm{kg\,s^{-1}})$
M	mass of agitated liquid (kg)
n	model parameter $(-)$
n	number of measurements $(-)$
N	impeller rotation speed $(\mathrm{s^{-1}})$
Nu	Nusselt number $(-)$
Pr	Prandtl number $(-)$
q	heat flux $(\mathrm{W\,m^{-2}})$
$\dot{Q}$	heat transfer rate (W)
r	model parameter $(-)$
Re	Reynolds number $(-)$
s	model parameter $(-)$
S	heat transfer area $(\mathrm{m^2})$
t	time (s)
T	temperature, temperature of agitated liquid $(^\circ\mathrm{C}, \mathrm{K})$
T_0	initial temperature of agitated liquid $(^\circ\mathrm{C}, \mathrm{K})$
T_{A}	temperature of heating or cooling liquid A $(^\circ\mathrm{C}, \mathrm{K})$
T'_{A}	inlet temperature of heating or cooling liquid A $(^\circ\mathrm{C}, \mathrm{K})$
T''_{A}	outlet temperature of heating or cooling liquid A $(^\circ\mathrm{C}, \mathrm{K})$
T_i	measured temperature of agitated liquid $(^\circ\mathrm{C}, \mathrm{K})$
T_{w}	wall temperature $(^\circ\mathrm{C}, \mathrm{K})$
$T_{\mathrm{w}i}$	wall temperature $(^\circ\mathrm{C}, \mathrm{K})$
Vi	viscosity ratio $(-)$
α	heat transfer coefficient $(\mathrm{W\,m^{-2}\,K^{-1}})$
δ_{w}	vessel wall thickness (m)
λ	thermal conductivity of agitated liquid $(\mathrm{W\,m^{-1}\,K^{-1}})$

λ_w	thermal conductivity of vessel wall $(\mathrm{W\,m^{-1}\,K^{-1}})$
μ	dynamic viscosity of agitated liquid $(\mathrm{Pa\,s})$
$\overline{\mu}$	dynamic viscosity of agitated liquid at mean temperature $(\mathrm{Pa\,s})$
μ_w	dynamic viscosity of agitated liquid at wall temperature T_W $(\mathrm{Pa\,s})$
ν	kinematic viscosity of agitated liquid $(\mathrm{m^2\,s^{-1}})$
ϱ	density of agitated liquid $(\mathrm{kg\,m^{-3}})$

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